13th Philippine Mathematical Olympiad

2011

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THE CONTEST

First held in 1984, the PMO was created as a venue for high school students with interest and talent in mathematics to come together in the spirit of friendly competition and sportsmanship. Its aims are: (1) to awaken greater interest in and promote the appreciation of mathematics among students and teachers; (2) to identify mathematically-gifted students and motivate them towards the development of their mathematical skills; (3) to provide a vehicle for the professional growth of teachers; and (4) to encourage the involvement of both public and private sectors in the promotion and development of mathematics education in the Philippines.

The PMO is the first part of the selection process leading to participation in the International Mathematical Olympiad (IMO). It is followed by the Mathematical Olympiad Summer Camp (MOSC), a five-phase program for the twenty national finalists of PMO. The four selection tests given during the process of MOSC determine the tentative Philippine Team to the IMO. The final team is determined after the third phase of MOSC.

The PMO this year is the thirteenth since 1984. Three thousand four hundred fifty-one (3451) high school students from all over the country took the Qualifying Stage examination. From this number, only two hundred nineteen (219) moved on to the Area Stage and now, in the National Stage, we are down to twenty who will compete for the top three positions and hopefully move on to represent the country in the 52nd IMO, which will take place in Amsterdam, Netherlands on July 16-24, 2011.

MESSAGE FROM DOST

The Department of Science and Technology - Science Education Institute (DOST-SEI) congratulates the Mathematics Society of the Philippines (MSP) for successfully conducting the 12th Philippine Mathematical Olympiad. Once again, the able members of MSP have put up a triumphant battle of wits and brains among high school students in the country.

We also congratulate the students who made it to the national finals of the PMO. You are the creme dela creme of your batch and being just in the finals makes you a winner already.

The silver medal the Philippines won at the 2010 International Mathematical Olympiad (IMO) held in Azerbaijan, Kazakhstan, as well as the honorable mentions that the rest of the team received, brought pride to our country. Recognized by President Aquino and the House of Representatives, the victory of the Philippine team to the 2010 IMO is a feat worth emulating. The standards are now raised higher for the winners of the 2011 PMO.

Now, the stakes are even higher as we embark on our journey to the 2011 IMO. The whole world is watching us as we have already shown what we are capable of doing. We are confident we can do better this year.

DOST-SEI will remain at the forefront of discovering new talents in science, technology, engineering and mathematics through the PMO and other competitions. We believe that providing the lamp posts through scholarships and mentoring programs for the student achievers will guide them into realizing their dreams of being part of the dynamic Philippine science community.

Moreover, DOST-SEI will continue to support programs that will uplift the status of mathematics education in the country through innovations in teacher education, human resource development and promotional activities to cull out talents in mathematics and create a culture of science.

Thank you and Mabuhay.

DR. ESTER B. OGENA
Director, DOST-SEI

MESSAGE FROM MSP

The Mathematical Society of the Philippines (MSP) has been at the forefront of the promotion of mathematics education and research in our country for 38 years. The MSP is proud to be part of the Philippine Mathematical Olympiad, the toughest and most prestigious math competition in the country. We are grateful to DOST-Science Education Institute for supporting the MSP in organizing this activity. The MSP and DOST-SEI are one in their objective of discovering and nurturing mathematical talents among the youth.

The Philippine Mathematical Olympiad brings together a number of the best high school students to show their natural talents and acquired knowledge in mathematics. These young people will surely contribute essentially to the creation of a bright future for our country.

In behalf of the MSP, I wish to thank the sponsors, schools and other organizations, institutions and individuals for their continued support and commitment to the PMO. Thank you and congratulations to Dr. Jose Ernie Lope and his team for the successful organization of the 13th PMO.

Congratulations to the winners and all the participants of the 13th PMO!

Jumela Sarmiento

DR. JUMELA F. SARMIENTO

President

Mathematical Society of the Philippines

MESSAGE FROM FUSE

It is a pleasure to be able to greet and congratulate you for winning recognition for the country and for yourself in a national prestigious Mathematics competition. Few earn such an honor in Mathematics.

As a student, I loved Mathematics. Although I can say I loved other subjects as much, I know Mathematics has helped me a lot in my life.

What makes your award rare is that not all students feel that way about Mathematics. Many even fear it. They should not feel that because Mathematics is the foundation of other sciences and disciplines.

In this recognition of your Mathematics ability, I advise you strongly, to keep cultivating your talent and doing your best at it. Improving every opportunity you have. Be inspired by the great mathematicians; they did not become great on single efforts but in sustaining their love and interest in this subject.

Congratulations! I wish you continuing success and honors.

DR. LUCIO C. TAN
Vice-Chairman
FUSE

MESSAGE FROM CASIO

It's another year to cheer for!

On behalf of CASIO COMPUTER CO., LTD. and Business Plus Marketing, the exclusive distributor of CASIO Calculators and its partner products in the Philippines, I am grateful to all the officers, organizers and all the people behind the 13th PHILIPPINE MATHEMATICAL OLYMPIAD especially University of the Philippines-Diliman and Mathematical Society of the Philippines, for trusting us again to participate in this one educational, essential and scholarly competition in the Mathematics field.

Business Plus Marketing (BPM) and its people are gratified that UP-Diliman together with the faculty, students and participants of the event, put their belief on CASIO. We believe that this year's PMO will have a successful and beneficial result to all of us. Looking forward to our strengthened relationship for educational dealings in the future.

More power and May God be with us all the time!

Yours truly,

JOEL C. SERRANO
Sales and Marketing Manager
Business Plus Marketing

MESSAGE FROM C&E

Like most ordinary mortals, I used to be frightened by numbers and other symbols that have to do with Mathematics. Such was my grade school days that, what could have been an early path to following after the footsteps of a CPA parent instead led to a "less mathematical" career by adulthood.

Which is not to say I have not since developed an appreciation for Math. Let's just say that like a beautiful woman who is way beyond my league, Mathematics is someone I could only love from a distance.

That is why, like gifts and love letters I could have secretly sent to this lady love of mine, it is with great pleasure that I am given this opportunity to contribute to the greater glory of Math by way of the Philippine Math Olympiad.

PMO's generosity to include C&E Publishing amongst its chosen benefactors is a testament to your organization's recognition of C&E's commitment to promoting knowledge towards academic and professional excellence. For this, we are forever grateful for your efforts to promote the Filipino students' interest in Mathematics.

My congratulations to the organizers, as well as to the students & parents, behind the annual PMO. I am sure that because of your relentless advocacy, more and more of our youth are finding "true love" in a subject that may have before eluded some but is now the opportunity for many in making the Philippines home to International Math Olympiad champions!

Mabuhay!

JOHN EMYL G. EUGENIO
VP – Sales & Marketing Division

SCHEDULE

Mathematical Society of the Philippines

PHILIPPINE MATHEMATICAL OLYMPIAD
NATIONAL STAGE

NISMED, UP Diliman | January 22, 2011

DOST-SEI · C&E Publishing, Inc. · F·U·S·E · CASIO CALCULATORS · Business Plus MARKETING

0730am - 0830am	Registration
0900am - 1200nn	Phase I - Written Phase
1200nn - 0200pm	Lunch Break
0200pm - 0500pm	Phase II - Oral Phase
	National Anthem
	Welcoming Remarks
	Awarding of Certificates
	Oral Competition
0630pm - 0830pm	Dinner and Awarding Ceremonies

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Mr. Crizaldy Binarao

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Dr. Jumar Valdez

Region 4A
Dr. Editha Jose

Region 4B
Engr. Elucila Sespeñe

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Mr. Lindley Kent Faina

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Saint Jude Catholic School

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Camille Tyrene Dee
Immaculate Conception Academy

Vance Eldric Go
Saint Jude Catholic School

Russelle Guadalupe
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Jamel Ramon Ibrahem
Magsaysay (Cubao) HS

Dongha Kang
Brent International School

Carmela Antoinette Lao
Saint Jude Catholic School

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Amiel Sy
Philippine Science HS - Main Campus

Joshua Uyheng
Bethany Christian School

Adrian Vidal
Philippine Science HS - Main Campus

Justin Yturzaeta
Jubilee Christian Academy

Part I.

Each correct answer is worth two points.

1. What is the sum of the roots of $x^2 - 2009x - 2010 = 0$?

   (a) 2010 (b) 2009 (c) 2011 (d) -2010

2. Find the value of $2\sqrt{2\sqrt{2\sqrt{2\sqrt{2\cdots}}}}$.

   (a) 2 (b) $\sqrt{2}$ (c) 4 (d) $2\sqrt{2}$

3. If $2^{2^x} = 4^3$, what is x?

   (a) $\log_2 6$ (b) $\log_4 6$ (c) $\log_6 2$ (d) $\log_6 4$

4. For what values of a does the system

$$\begin{cases} x^2 - y^2 &= 0 \\ (x-a)^2 + y^2 &= 0 \end{cases}$$

   have a unique solution?

   (a) $a = -1$ (b) $a = 0$ (c) $a = 1$ (d) $a = 2$

5. If $x + y = 4$ and $x^2 + y^2 = 10$, what is the value of $x^4 + y^4$?

   (a) 84 (b) 100 (c) 68 (d) 82

6. Let f be a function defined on the set of integers such that $f(1) = 5$ and $f(x+1) = 2f(x) + 1$ for all integers x. What is the value of $f(7) - f(0)$?

   (a) 380 (b) 189 (c) 191 (d) 381

7. There are k zeros at the end of $34! = 34 \cdot 33 \cdot 32 \cdot \cdots \cdot 4 \cdot 3 \cdot 2 \cdot 1$. What is the value of k?

   (a) 7 (b) 4 (c) 6 (d) 5

8. Find the sum $\cos 1^\circ + \cos 3^\circ + \cos 5^\circ + \cdots + \cos 177^\circ + \cos 179^\circ$.

   (a) $\frac{\sqrt{2}}{2}$ (b) 1 (c) 0 (d) $\frac{1}{2}$

9. If $\frac{18x+7y}{12y+5x} = \frac{2}{3}$, what is the value $\frac{x}{y}$?

   (a) $\frac{57}{46}$ (b) $\frac{44}{3}$ (c) $\frac{46}{57}$ (d) $\frac{3}{44}$

10. A 4 by 6 inch paper is folded so that its upper right corner touches the midpoint of an opposite side and such that the fold obtained is the longer one. Find the length of the fold.

    (a) $2\sqrt{13}$ in (b) 5 in (c) $\sqrt{65}$ in (d) $5\frac{5}{24}$ in

11. If $a - b + c = 1$, $b - 2c = 0$, $2a + c = 5$, what is the sum $a + b + c$?

    (a) 3 (b) 4 (c) 5 (d) 0

12. A triangle is formed inside a circle by connecting the center C to two points A and B on the circle. If $\angle ACB = 30^\circ$, what is the ratio of the areas of the circle to the triangle?

    (a) $6\pi : 1$ (b) $9 : 1$ (c) $4\pi : 1$ (d) $9\pi : 2$

13. A ball rebounds each time to a height which is half that of the previous one. If the total distance traveled before coming to rest is 72 meters, from how high was the ball dropped?

    (a) 24 meters (b) 18 meters (c) 36 meters (d) 12 meters

14. Let f be the function defined by $f(x) = \frac{\pi^x + \pi^{-x}}{\pi^x - \pi^{-x}}$. Find $f(2p)$ if $f(p) = 2$.

    (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) 4

15. If $x > 0$, find the solution set of $\log x \geq \log 2 + \log(x - 1)$.

    (a) $(1, 2]$ (b) $(-\infty, 2]$ (c) $(0, 1]$ (d) $(\sqrt{2}, 1]$

Part II.

Each correct answer is worth three points.

1. Solve for (x, y) in the system $(e^x + 2)^2 - y = 3$, $4(e^x + 2) - y = -1$.

   (a) $(\sqrt{2}, 3)$ (c) $(\ln \sqrt{2}, 3)$

   (b) $(\ln 2\sqrt{2}, 9 + 8\sqrt{2})$ (d) $(\ln \sqrt{2}, 2 + 4\sqrt{2})$

2. Mica has six differently colored crayons. She can use one or more colors in her painting. What is the likelihood that she will use only her favorite color?

   (a) $\frac{1}{24}$ (b) $\frac{1}{48}$ (c) $\frac{1}{81}$ (d) $\frac{1}{63}$

3. If $b_1 = \frac{1}{3}$ and $b_{n+1} = \frac{1 - b_n}{1 + b_n}$, for $n \geq 2$, find $b_{2010} - b_{2009}$.

   (a) $\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$

4. Let

$$x = 1 - \cfrac{1}{2 - \cfrac{1}{1 - \cfrac{1}{2 - \cfrac{1}{1 - \cdots}}}}.$$

   Find $(2x - 1)^2$.

   (a) 4 (b) -4 (c) 8 (d) -8

5. $\cos 15^\circ$ is equal to

   (a) $\sqrt{\frac{2 - \sqrt{3}}{2}}$ (b) $\sqrt{\frac{2 - \sqrt{3}}{4}}$ (c) $\frac{\sqrt{6} - \sqrt{2}}{4}$ (d) $\frac{\sqrt{6} + \sqrt{2}}{4}$

6. Solve for x in the equation $\frac{(\log_5 x)^2 - 4}{(\log_5 x)^2 + \log_5 x^4 + 4} + 2\log_5 x = -1$.

   (a) $x = 1$ (b) $x = -1$ (c) $x = 2$ (d) $x = 3$

7. A line with y-intercept 5 and positive slope is drawn such that this line intersects $x^2 + y^2 = 9$. What is the least slope of such a line?

   (a) $\frac{1}{3}$ (b) 1 (c) $\frac{5}{6}$ (d) $\frac{7}{6}$

8. A metal bar bent into a square is to be painted. How many distinct ways can one color the metal bar using four distinct colors on the edges using red, white, blue, and yellow.

   (a) 8 (b) 24 (c) 3 (d) 4

9. If $9^{2x} - 9^{2x-1} = 8\sqrt{3}$, find $(2x-1)^{2x}$.

   (a) $\frac{\sqrt{2}}{8}$ (b) $\frac{\sqrt{2}}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

10. In how many ways can the letters of the word MURMUR be arranged without letting two letters which are the same be adjacent?

    (a) 54 (b) 24 (c) 45 (d) 36

Part III.

Each correct answer is worth six points.

1. Let

$$f(n) = \begin{cases} n+1, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$$

   be a function whose domain is the set of positive integers. Then $f\left((n^2+1)^2 + (n^2-1)^2\right) =$

   (a) $2n^4 - 1$ (b) $2n^4$ (c) $2n^4 + 1$ (d) $2n^4 + 2$

2. Find all polynomials $p(x)$ where $xp(x-1) = (x-5)p(x)$ and $p(6) = 5!$

   (a) $\left\{\frac{x(x-1)(x-2)(x-3)(x-4)(x-5)}{6},\ 120x\right\}$

   (b) $\left\{\frac{x(x-1)(x-2)(x-3)(x-4)}{6}\right\}$

   (c) $\{x(x-1)(x-2)(x-3)(x-4)\}$

   (d) $\left\{\frac{x(x-1)(x-2)(x-3)(x-4)}{6},\ 24x\right\}$

3. Let $n = 2^{31}3^{19}$. How many positive divisors of n^2 are less than n but do not divide n?

   (a) 588 (b) 560 (c) 561 (d) 589

4. Four spheres, each of radius 1.5, are placed in a pile with three at the base and the other on top. If each sphere touches the other three spheres, give the height of the pile.

   (a) $3 + \sqrt{3}$ (b) $3 + \sqrt{6}$ (c) $\sqrt{6}$ (d) $6\sqrt{3}$

5. Let ABC be a 3-digit number such that its digits A, B, and C form an arithmetic sequence. The largest integer that divides all numbers of the form $ABCABC$ is

   (a) 11 (b) 101 (c) 1001 (d) 3003

Part I.

No solution is needed. All answers must be in simplest form.
Each correct answer is worth three points.

1. Find the solution set to the equation $(x^2 - 5x + 5)^{x^2-9x+20} = 1$.
2. Suppose $x(x - b - 3) = -2(b + 1)$. Find x.
3. The quotient of the sum and difference of two integers is 3, while the product of their sum and difference is 300. What are the integers?
4. Find the last 2 nonzero digits of 16!
5. Let $f(x)$ be a cubic polynomial. If $f(x)$ is divided by $2x + 3$, the remainder is 4, while if it is divided by $3x + 4$, the remainder is 5. What will be the remainder when $f(x)$ is divided by $6x^2 + 17x + 12$?
6. The operation $*$ satisfies the following properties:

$$x * 0 = 0, \;\; x * (y + 1) = x * y + (x - y).$$

   Evaluate $2010 * 10$.
7. Find the probability of obtaining two numbers x and y in the interval $[0, 1]$ such that $x^2 - 3xy + 2y^2 > 0$.
8. Find all complex numbers x satisfying $x^3 + x^2 + x + 1 = 0$.
9. Find the range of the function $f(x) = 2^{x^2-4x+1}$.
10. A "fifty percent mirror" is a mirror that reflects half the light shined on it back and passes the other half of the light onward. Now, two "fifty percent mirrors" are placed side by side in parallel and a light is shined from the left of the two mirrors. How much of the light is reflected back to the left of the two mirrors?
11. Find the sum of the coefficients of the polynomial $\cos(2 \arccos(1 - x^2))$.
12. Let $s_1 = 2^{2010}$. For $n > 2$, define

$$s_{n+1} = \begin{cases} \log_{\sqrt{2}} s_n, & s_n > 0 \\ 0, & s_n \leq 0 \end{cases}$$

    Find the smallest n such that $s_n \in [4, 6]$.
13. Two students, Lemuel and Christine, each wrote down an arithmetic sequence on a piece of paper. Lemuel wrote down the sequence $2, 9, 16, 23, ...$, while Christine wrote down the sequence $3, 7, 11, 15, ...$ After they have both written out 2010 terms of their respective sequences, how many numbers have they written in common?

14. The line from the origin to the point $(1, \tan 75^\circ)$ intersects the unit circle at P. Find the slope of the tangent line to the circle at P.

15. Let $f(x)$ be a nonzero function whose domain and range is the set of complex numbers. Find all complex numbers x such that
$f(x^2) + xf\left(\dfrac{1}{x^2}\right) = \dfrac{1}{x}$.

16. Consider addition $\oplus$ and multiplication $\otimes$ modulo 7 of the numbers in $S = \{0, 1, 2, 3, 4, 5, 6\}$. This means that

$$m \oplus n = \text{remainder when } m + n \text{ is divided by } 7$$

$$m \otimes n = \text{remainder when } m \times n \text{ is divided by } 7.$$

Then 1 is the multiplicative identity and each element $a \in S$ has a multiplicative inverse $\dfrac{1}{a}$. Find the value of $\dfrac{1}{4} \oplus \left(2 \otimes \dfrac{1}{3}\right)$ in this number system.

17. Find all real numbers a such that $x^3 + ax^2 - 3x - 2$ has two distinct real zeros.

18. A circle with center C and radius r intersects the square $EFGH$ at H and at M, the midpoint of EF. If C, E and F are collinear and E lies between C and F, what is the area of the region outside the circle and inside the square in terms of r?

19. What is the remainder when $(0! + 1! + 2! + \cdots + 2011!)^2$ is divided by 10?

20. Let $a = 444\cdots444$ and $b = 999\cdots999$ (both have 2010 digits). What is the $2010th$ digit of the product ab?

Part II.

Show the solution to each item. Each complete and correct solution is worth ten points.

1. Sherlock and Mycroft play a game which involves flipping a single fair coin. The coin is flipped repeatedly until one person wins. Sherlock wins if the sequence TTT (tails-tails-tails) shows up first while Mycroft wins if the sequence HTT(heads-tails-tails) shows up first. Who among the two has a higher probability of winning?

2. Denote by a, b and c the sides of a triangle, opposite the angles α, β and γ, respectively. If α is sixty degrees, show that $a^2 = \dfrac{a^3 + b^3 + c^3}{a + b + c}$.

3. Show that $\sqrt[n]{2} - 1 \leq \sqrt{\dfrac{2}{n(n-1)}}$ for all positive integers $n \geq 2$.

Qualifying Stage

Test I			Test II		Test III
1. B	6. D	11. C	1. B	6. A	1. C
2. C	7. C	12. C	2. D	7. $\frac{4}{3}$**	2. B
3. A	8. A	13. A	3. C	8. C	3. D
4. B	9. C	14. C	4. -1*	9. A	4. B
5. D	10. D	15. A	5. D	10. 30**	5. D

* The question was discarded since the given continued fraction is divergent. If the question of convergence is not taken into account, the correct answer would have been -1.
* This question was discarded because the correct answer was not among the choices.

Area Stage

Test I

1. $\{1, 2, 3, 4, 5\}$
2. $x = b + 1$ or $x = 2$
3. $(20, 10), (-20, -10)$
4. 88
5. $6x + 13$
6. $20,055$
7. $\frac{3}{4}$
8. $x = -1, i, -i$
9. $\left[-\frac{1}{8}, \infty\right)$
10. $\frac{2}{3}$
11. -1
12. 6
13. 287
14. $-2 + \sqrt{3}$
15. item was scrapped
16. 5
17. $a = 0$
18. $r^2\left(\frac{22}{25} - \frac{tan^{-1}\frac{4}{3}}{2}\right)$
19. 6
20. 3

Test II

1. Sherlock has probability $\frac{1}{8}$ of winning while Mycroft has probability greater than $\frac{1}{8}$. The event that Sherlock wins is the set $\{TTT\}$ hence $P(\{TTT\}) = \frac{1}{8}$ while the event that Mycroft wins is the set $\{HTT, HHTT, THTT, HHHTT, TTHTT, HTHTT, THHTT, \ldots\}$ and $P(M) > \frac{1}{8}$.
2. By Cosine Law, $a^2 = b^2 + c^2 - 2bc\left(\frac{1}{2}\right)$. $b^3 + c^3 = (b + c)(b^2 - bc + c^2) = (b + c)a^2$. Add a^3 to both sides and move terms to get the desired equation. That is, $a^3 + b^3 + c^3 = (b + c)a^2 + a^3 = (a + b + c)a^2$ and the desired inequality follows.
3. Let $x_n = \sqrt[n]{2} - 1 \geq 0$. Then $2 = (1 + x_n)^n \geq 1 + nx_n + \frac{n(n-1)}{2}x_n^2 \geq 1 + \frac{n(n-1)}{2}x_n^2$. Thus $\frac{n(n-1)}{2}x_n^2 \leq 2 - 1 = 1$ and the desired probability follows.

PMO: THROUGH THE YEARS

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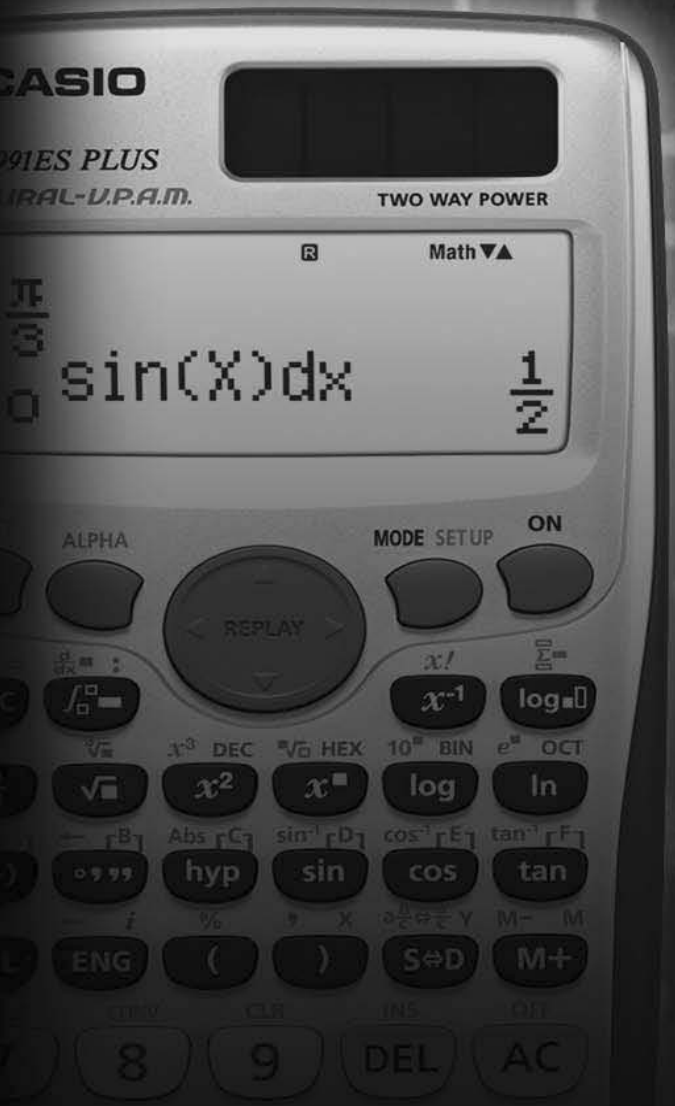

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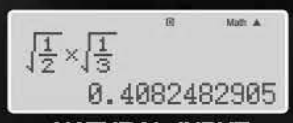
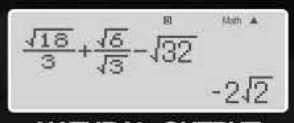


NATURAL INPUT
Input expressions and arithmetic operations as they appear in written form.



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