THE 25TH PHILIPPINE MATHEMATICAL OLYMPIAD

25th

PHILIPPINE MATHEMATICAL OLYMPIAD

Qualifying Stage

January 14, 2023
Various Test Centers Nationwide

Area Stage

February 11, 2023
Various Test Centers Nationwide

National Stage

March 18 & 19, 2023
Institute of Mathematics
University of the Philippines Diliman

Awarding Ceremony

March 19, 2023
SOLAIR Auditorium
University of the Philippines Diliman

The 25th Philippine Mathematical Olympiad is a project of the Department of Science and Technology - Science Education Institute and the Mathematical Society of the Philippines.

The 25th PMO is also made possible by the support of our partners.

About the PMO

First held in **1984**, the **Philippine Mathematical Olympiad (PMO)** was created as a venue for high school students with interest and talent in mathematics to come together in the spirit of friendly competition and sportsmanship. Its aims are:

1) to stimulate the improvement of mathematics education in the country by awakening greater interest in and appreciation of mathematics among students and teachers, and gaining insights into the levels of mathematical learning;
2) to identify and motivate the mathematically gifted;
3) to identify potential participants to the International Mathematical Olympiad;
4) to provide a vehicle for the professional growth of teachers; and
5) to encourage the involvement of both public and private sectors in the concerted promotion and development of mathematics education.

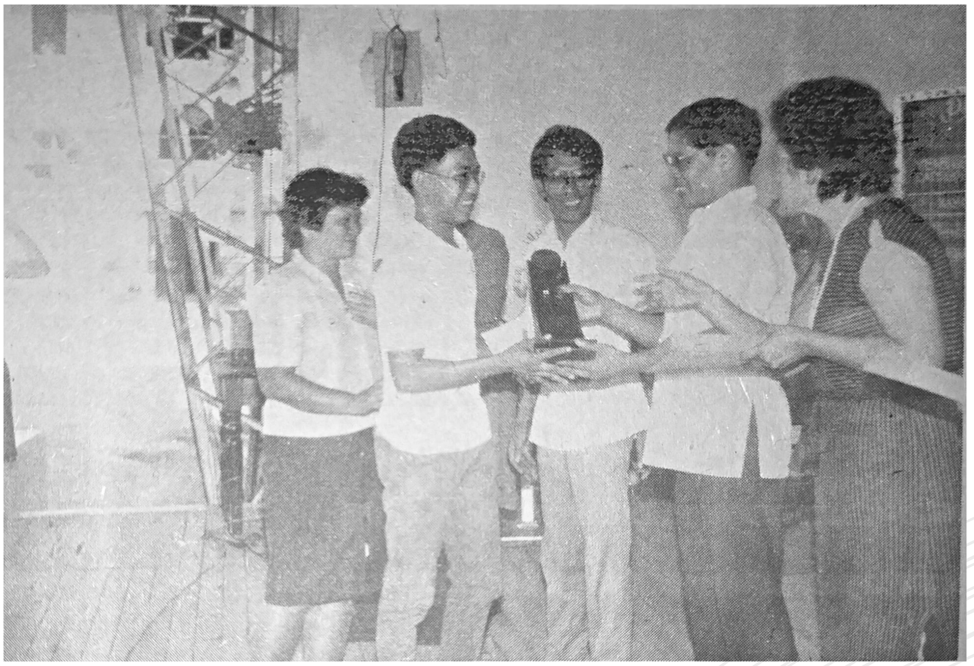
The PMO is only the first part of the selection program implemented by the Mathematical Society of the Philippine towards the country's participation in the **International Mathematical Olympiad (IMO)**. The twenty national finalists of the PMO will be invited to the **Mathematical Olympiad Summer Camp (MOSC)**, a training program where participants will experience problem solving at a level that will help them grow in mathematical maturity, in preparation for the IMO. The selection tests and quizzes will then determine the six contestants who will form the country's National Team in Mathematics - the Philippine Team to the International Mathematical Olympiad.

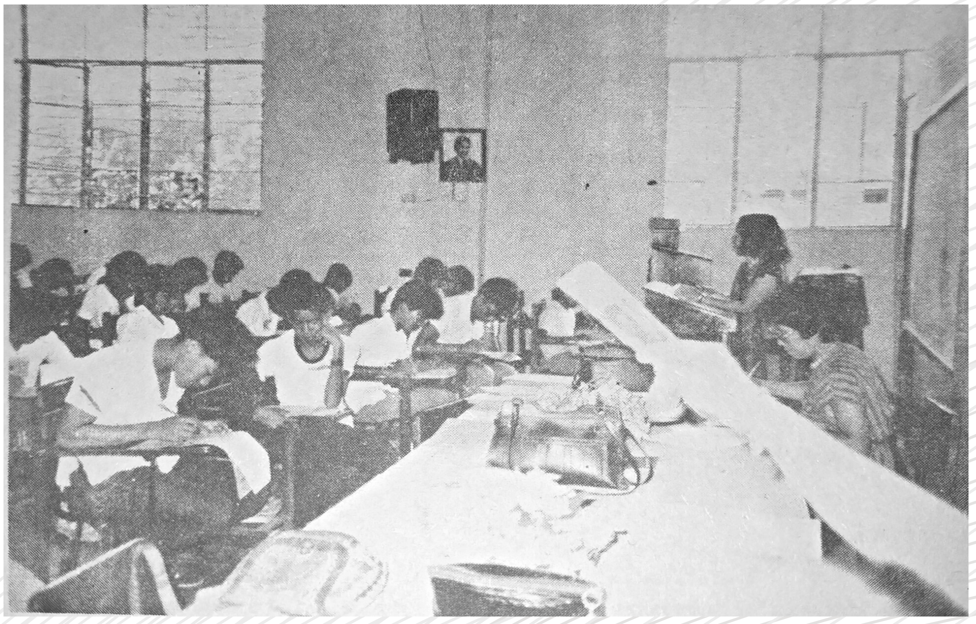
The Philippine Mathematical Olympiad, the Mathematical Olympiad Summer Camp, and the country's participation in the International Mathematical Olympiad, are projects of both the **Mathematical Society of the Philippines** and the **Department of Science and Technology - Science Education Institute**.

The PMO this year is the twenty-fifth since 1984. After two years of fully online competitions, contestants this year were able to **compete onsite** (in-person) in 35 test centers nationwide. We were also able to return to the 3-stage structure of the competition by holding the Area Stage again. A silver lining from our experience over the past two years is the PMO being able to accommodate contestants who could only compete online. Last January, **4,531 contestants** from Grades 7 to 12 joined the Qualifying Stage. From this number, **180** joined the Area Stage last February. Our **20 National Finalists** were then selected from this pool. We will choose from this group the Philippine Team to the **64th International Mathematical Olympiad** which will be held from **July 2 to 13** in **Chiba, Japan**.

PMO Past Years

PMO	School Year
Trial Run	1984-1985
1st	1986-1987
2nd	1988-1989
3rd	1990-1991
4th	1992-1993
5th	1994-1995
6th	1996-1997
7th	1998-1999
8th	2000-2001
9th	2003-2004
10th	2007-2008
11th	2008-2009
12th	2009-2010
13th	2010-2011
14th	2011-2012
15th	2012-2013
16th	2013-2014
17th	2014-2015
18th	2015-2016
19th	2016-2017
20th	2017-2018
21st	2018-2019
22nd	2019-2020
23rd	2020-2021
24th	2021-2022
25th	2022-2023





Photographs taken from the report for the 1984-1985 trial run of the PMO

Past Messages

REPUBLIC OF THE PHILIPPINES
NATIONAL SCIENCE AND TECHNOLOGY AUTHORITY
SCIENCE PROMOTION INSTITUTE

M E S S A G E

THE SUCCESSFUL CONDUCT OF THE 1985 PHILIPPINE MATH OLYMPIAD DENOTES OF THE IMPROVING QUALITY OF MATHEMATICS EDUCATION IN THE COUNTRY, AND OF THE COMPARATIVELY TALENTED YOUNG BOYS AND GIRLS IN OUR MIDSTS.

IT IS A DELIGHTFUL EXPERIENCE TO SEE YOUNG MINDS ENGAGED IN SUCH INTELLECTUAL PROWESS, SOLVING PROBLEMS WE HAVE NOT THOUGHT THEM CAPABLE OF DEALING WITH. IT IS INDEED AMAZING WHAT THEY CAN ACHIEVE, IF PROPERLY CHALLENGED.

ACTIVITIES SUCH AS THIS ENHANCE INGRAINED POSITIVE QUALITIES AMONG OUR YOUTHS, AND DEVELOP THEIR INTELLECTUAL SKILLS AND ABILITIES NEEDED TO COPE WITH CHANGE.

MY SINCERE CONGRATULATIONS TO THE ORGANIZERS AND COOPERATORS OF THIS BENEFICIAL UNDERTAKING.

JOSE L. GUERRERO
DIRECTOR

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MATHEMATICAL SOCIETY OF THE PHILIPPINES

M E S S A G E

The Philippine Mathematical Olympiad is one of the latest, one of the biggest and one of the most exciting projects of the MSP. In line with the general objective of the Society to improve mathematics education in the country, the MSP is one of the main sponsors of the PMO.

The Society is already engaged in efforts to upgrade mathematics eduation at the tertiary level. In this connection there is a project to formulate and disseminate model mathematics curricula, complete with course outlines, textbook and reference lists, and sample examinations. For the benefit of teachers, there are also regular lectures on advanced topics in mathematics. There are seminars in which results of original researches done by Filipino mathematicians are reported.

The MSP's sponsorship of the PMO, which is geared toward secondary level students, is a natural complement to all the Society's other efforts addressed to higher level mathematics education and research.

The Society hopes that the Olympiad, specially when it is implemented on a national scale in 1986-87, will provide a special incentive for students to study harder and excel in mathematics, for their teachers to improve the quality and the content of their teaching, and for school administrators to provide the appropriate support and inducement for their students and teachers. For the the policy makers in education, the results of the PMO will be a rich source of data that can be used for better monitoring, planning and implementation in the teaching of mathematics among our high schools.

All the officers and members of the MSP have worked hard and pledge to continue working hard for the Philippine Mathematical Olympiad. We are all proud to have taken part in launching what could be an important tradition in our country.

HONESTO G. NUGUI
President

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Messages taken from the report for the 1984-1985 trial run of the PMO

Message

Cheers to 25 years!

Mathematics plays an indispensable role in shaping society. From building bridges and communities to maintaining fairness in elections, mathematics is undoubtedly involved in all spheres of life. Hence, understanding the concept of mathematics is vital because irrespective of what career one chooses to pursue, the effect of mathematics will be felt inevitably. And students who are especially interested in pursuing a career in science, technology, engineering, and mathematics (STEM) should understand that a strong grasp of numbers is essential to realize their college dreams.

Recreational math competitions offer an effective avenue to help hone students' numerical talent in a challenging, yet fun and exciting environment. Such is the Philippine Mathematical Olympiad (PMO), the oldest and most reputable local math olympiad in the country developed and progressed by the Mathematical Society of the Philippines (MSP) and the Department of Science and Technology-Science Education Institute.

For 25 years, the PMO has flourished to become one of the most sought-after academic competitions in the country, aiming to nurture high school students' interest and appreciation for mathematics. And after two years of holding the contests virtually, the campaign to transform conventional math learning strategies into a more engaging one continues as the PMO successfully returns to its in-person activities, with even more test centers than before the pandemic. Such quick transitions from face-to-face to online and vice versa have been accompanied by many challenges, but nothing beats the passion, commitment, and determination of the people behind the competition to prove that the Filipino youth are equally capable of competing in mathematical tournaments.

Considering this mission, we, at the DOST-SEI, celebrate our partner MSP as it celebrates its 50th founding anniversary and we pledge to continue doing what we can to help them in providing the Filipino students with ample opportunities to cultivate their love, passion, and enthusiasm for mathematics, encourage their inner competitive character, and build confidence in their potential. Trust that we will strive harder to become the breeding ground for our country's future STEM professionals and STEM advocates for greater positive change.

As we rejoice in the success of the 25th PMO, we also remind everyone that winning is not about finishing in first place or beating the others, rather, it is surpassing oneself and overcoming one's fears, weaknesses, and limitations. So, no matter how the competition ended, jump, celebrate, and acknowledge your efforts. Raise your glass and remember the journey!

JOSETTE T. BIYO

Director, Science Education Institute
Department of Science and Technology

Message

Congratulations to all the participants of the 25th edition of the Philippine Mathematical Olympiad, most especially to the national finalists and winners! Congratulations also to the PMO Organizing Team led by Dr. Richard Eden, all the Regional Coordinators, and the Test Development Committee headed by Dr. Christian Paul Chan Shio, for the successful conduct of the Olympiad, held mostly on-site.

Just like last year, we have finalists from Luzon, Visayas and Mindanao. We also have two female finalists. This is a welcome trend, which we hope will persist in the succeeding editions of the PMO.

Although the first PMO was held in the 1980s, this year is only the 25th PMO, as during its early years, the Olympiad was held only once every two years. Thanks to the support of the DOST-SEI and of our sponsors, we are now able to hold the Olympiad every year. The Mathematical Society of the Philippines is committed to nurturing young Filipina and Filipino math enthusiasts, who we hope will eventually pursue careers in mathematics and its allied fields.

The PMO is several notches above other math competitions held in the country mainly because of the quality of its questions, which are formulated by veteran math competitors and designed to identify potential members of the Philippine Team. The MSP is most certainly pleased with our Team's performance in the Asia Pacific and International Mathematical Olympiads in recent years, and we are confident that our Team can replicate our recent accomplishments.

The MSP is also celebrating a milestone this year — its 50th anniversary! Since its foundation in 1973, it has provided avenues for its members and for the larger Philippine mathematical community to pursue their research interests, disseminate their findings, and further enhance their mathematical capabilities. I hope that some of our PMO contestants will continue their pursuit of mathematics and become members of the MSP in the future.

Padayon!

JOSE ERNIE C. LOPE
President
Mathematical Society of the Philippines

Message

Greetings and congratulations to the Philippine Mathematical Olympiad on its 25th year — this is indeed a milestone!

Congratulations, too, to those who made it to the National Stage! You have evidenced your elite talent in mathematics. Additionally, you have developed discipline and perseverance which are lifelong skills that will be beneficial to you in your young life. As in any competition, there are winners that emerged and when you did, you realized that the mathematical problems you were given were not easy but you worked hard on them employing skills and strategies as you were under time constraint. There was also, maybe a problem or two that you were unable to solve but that made you realize you have limitations leading to humility. On a social level, PMO as a friendly competition gave you the opportunity to meet new friends and feel kindness for each one in all the stages that you went through.

Congratulations and thank you to the coaches — your participation in the Olympiad enabled you to share your passion for mathematics to young minds. It is this passion for all involved in PMO that will ensure the next 25 years of fruitful achievement!

LUCIO C. TAN
Chairman
Foundation for Upgrading the
Standard of Education Inc.

Message

CASIO

On behalf of Casio Education Philippines, We are so proud of those students who represented the schools at this year's Philippine Math Olympiad!

The awards and medals are proudly witnessing your hard work and best dedication. The award makes you shining in midst of names and fame. Keep dreaming in your life and reap all your dreams with your hard work. Congratulation for your achievement.

JOEL C. SERRANO

Sales and Marketing Manager
Casio Education Philippines
Marius Holdings Corporation

Message

C&E Adaptive Learning Solutions (C&E ALS) extends our warmest congratulations to the Mathematical Society of the Philippines for the continuous success of the Philippine Mathematical Olympiad over recent years.

With this, we also extend our sincere greetings as the Philippine Mathematical Olympiad celebrates its silver anniversary, a testament to the society's dedication to fostering excellence in mathematics. May this milestone achievement continue to inspire math achievers nationwide. May the society's initiatives continue to bear fruit and pave the way to achieving even greater heights of success in the years to come.

C&E ALS reaffirms our support for both the society and the mathematically inclined students in their endeavor to be in the global rankings for mathematics. Through the contributions of C&E ALS, we hope to support the advancement of math abilities among the mathematically gifted as they engage in healthy competition to achieve excellence.

JOHN EMYL G. EUGENIO
Chief Operations Officer
C&E Adaptive Learning Solutions

The PMO Team

EXECUTIVE COMMITTEE

Director	Richard Eden
Associate Director	Daryl Allen Saddi
Secretary	Rheadel Fulgencio

ASSISTANT DIRECTORS

Qualifying Stage	John Vincent Morales
Area Stage	Luis Silvestre Jr.
National Stage	Mark Jason Celiz

OPERATIONS

Treasurer	Calvin Sia
Project Assistant	Gaudelia Ruiz
Designer	Samantha Dellomas
Webpage Manager	Nathaniel Pereda

TEST DEVELOPMENT COMMITTEE

Head	Christian Paul Chan Shio
Critics	Louie John Vallejo Carl Joshua Quines
Members	Carlo Francisco Adajar Sarji Elijah Bona Russelle Guadalupe Lu Christian Ong Sean Anderson Ty Kerish Villegas

REGIONAL COORDINATORS

Region I & CAR	Shielden Grace Domilies
Region 2	Crizaldy Binarao
Region 3	Yolanda Roberto
Region 4A	Sharon Lubag
Region 4B	Emmalyn Venturillo
Region 5	Francis Delloro
Region 6	Alexander Balsomo
Region 7	Cherrylyn Alota
Region 8	Oreste Ortega, Jr.
Region 9	Dante Partosa
Region 10	Paolo Araune
Region 11	Joseph Belida
Region 12, 13, & BARMM	Miraluna Herrera
NCR	Kristine Joy Carpio Jude Buot

TECHNICAL STAFF

Members	Vitus Paul de Jesus Ralph Joshua Macarasig Joyce Tercero Carmi Villarama

The PMO Team

SATELLITE TEST CENTER COORDINATORS

Region I & CAR Celestino Jerome Picar

Region 2 Whilmar Villanueva, Roquito Panit

Region 4A Karen Nocum, Jane Palacio, Amelia Jarapa

Region 4B Jethro Cullen Sia, Christopher Lao

Region 5 Edgar Arciga, Rowan Celestra

Region 6 Jahfet Nabayra

Region 9 Maria Wendy Parojinog, Jennifer Cardente

Region 10 Mhelmar Labendia, Sharon Hubahib Esporsado

Region 12, 13, & BARMM Maria Montserrat Magdael, Berlita Disca, Gheleene Olayvar Sering-Buenaflor

The PMO is grateful to these schools which served as test centers.

Region I & CAR
Benguet State University
La Union Cultural Institute

Region 2
Cagayan State University
Cabaroguis National School of Arts and Trades
Maddela Comprehensive High School

Region 3
Bulacan State University

Region 4A
De La Salle University - Dasmariñas
University of Batangas
University of the Philippines Los Baños
Calayan Education Foundational Inc.

Region 4B
Western Philippines University
Landy National High School

Region 5
Ateneo de Naga University
Legazpi City Science High School
Sorsogon National High School

Region 6
West Visayas State University
Aklan State University

Region 7
University of the Philippines Cebu
Mandaue City Science High School
Naga National High School

Region 8
Leyte Normal University

Region 9
Ateneo de Zamboanga University
Basilan National High School
Zamboanga Sibugay National High School

Region 10
Xavier University - Ateneo de Cagayan
Mindanao State University - Iligan Institute of Technology
Misamis Occidental National High School

Region 11
Ateneo de Davao University

Region 12, 13, and BARMM
Caraga State University
Notre Dame University
Mindanao State University - General Santos
St. Paul University Surigao

NCR
Ateneo de Manila University
De La Salle University
University of the Philippines Diliman
Philippine Science High School - Main Campus
British School Manila

Awards

TOP CONTESTANT PER REGION
IN THE QUALIFYING STAGE

NCR — **Jerome Austin N. Te**
Jubilee Christian Academy

Region 1 — **Jairus Carle C. Isaguirre**
Philippine Science High School
- Ilocos Region Campus

CAR — **Harley Ty**
Philippine Science High School
- Cordillera Administrative Region Campus

Region 2 — **Sean Kyro B. Patino**
University of Saint Louis

Region 3 — **Justin M. Sarmiento**
Marcelo H. Del Pilar National High School

Region 4A — **Gadriel Symone R. Dalangin**
Philippine Science High School
- CALABARZON Region Campus

Region 4B — **Krishna Reign O. Delos Reyes**
Porfirio G. Comia Memorial National High School

Region 5 — **Jose Carlo C. Salazar**
Philippine Science High School
- Bicol Region Campus

Region 6 — **Jonathan D. Anacan**
Philippine Science High School
- Western Visayas Campus

Region 7 — **Jose Mari Paolo B. Rollan**
City of Bogo Senior High School

Region 8 — **Karl Louie G. Laude**
Philippine Science High School
- Eastern Visayas Campus

Region 9 — **Hans Cedric L. Galande**
Zamboanga Chong Hua High School

Region 10 — **Mohammad Nur G. Casib**
Philippine Science High School
- Central Mindanao Campus

Region 11 — **Leonardo Florenz S. Eugenio**
Philippine Science High School
- Southern Mindanao Campus

Region 12 — **Felinwright Niñokyle A. Mesias**
Philippine Science High School
- SOCCSKSARGEN Region Campus

Region 13 — **Joseph T. Banaybanay**
Bayugan National Comprehensive High School

BARMM — **Bai Zharean Jathiya S. Ante**
Albert Einstein School, Inc.

Shahanie A. Ditucalan
Mindanao State University
- Marawi Senior High School

Joharima P. Grande
RC - Al Khwarizmi International College Foundation Inc. Science Laboratory School

Awards

AREA STAGE WINNERS

LUZON

1st Justin M. Sarmiento
Marcelo H. Del Pilar National High School

2nd Benedict S. Rodil
Dasmariñas Integrated High School

3rd Rainier M. Guinto
Cavite Science Integrated School

VISAYAS

1st Jonathan D. Anacan
Philippine Science High School - Western Visayas Campus

2nd Lance Christopher M. Jimenez
San Roque College De Cebu

3rd Jose Mari Paolo B. Rollan
City of Bogo Senior High School

MINDANAO

1st Mohammad Nur G. Casib
Philippine Science High School - Central Mindanao Campus

2nd Leonardo Florenz S. Eugenio
Philippine Science High School - Southern Mindanao Campus

Felinwright Niñokyle A. Mesias
Philippine Science High School - SOCCSKSARGEN Region Campus

NCR

1st Raphael Dylan T. Dalida
Philippine Science High School - Main Campus

Filbert Ephraim S. Wu
Victory Christian International School

3rd Alvann Walter W. Paredes Dy
Saint Jude Catholic School

Jerome Austin N. Te
Jubilee Christian Academy

Awards

SPECIAL AWARDS

The 25th PMO will hand out the following special awards at the Awarding Ceremony.

Top Junior Contestant

This is for the student from Grade 7, Grade 8, or Grade 9 with the highest score in the National Stage. The awardee will receive a medal and a certificate.

Top Female Contestant

This is for the female contestant with the highest score in the National Stage. The awardee will receive a medal, certificate, and P 5,000 from For the Women Foundation.

PRIZES FOR PMO WINNERS

The prizes for the three winners of the PMO in the National Stage are the following:

CHAMPION **P 25,000, medal, trophy, certificate**

FIRST RUNNER-UP **P 20,000, medal, trophy, certificate**

SECOND RUNNER-UP **P 15,000, medal, trophy, certificate**

Their coaches will receive P 5,000, P 3,000, and P 2,000, respectively, and a certificate. Their schools will receive a trophy.

All National Finalists will be invited to the Mathematical Olympiad Summer Camp. The MOSC is the training camp that will select the country's contestants to the 64th International Mathematical Olympiad this July 2023 in Chiba, Japan.


REGION 02

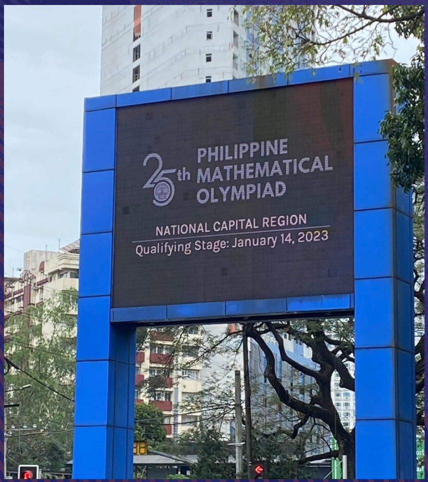

Philippine Mathematical
Olympiad
PMO
Welcome participants to the
QUALIFYING
STAGE
Sorsogon National High School
Sorsogon City
(Satellite Testing Center)


Welcome Participants, Coaches, and Guests!
PHILIPPINE
MATHEMATICAL
OLYMPIAD
REGION III
Qualifying Stage: January 14, 2023
Area Stage: Febuary 11, 2023


OLYMPIAD
Region 12, BARMM & Caraga
Qualifying Stage: January 14, 2023
Area Stage: February 11, 2023
cebu pacific


GREEN ARCHER STATUE
25PMO


Welcome Participants, Coaches, and Guests!
25th PHILIPPINE MATHEMATICAL OLYMPIAD
REGION VI (WESTERN VISAYAS)
Qualifying Stage: January 14, 2023
Area Stage: February 11, 2023
cebu pacific
CASIO


25th PHILIPPINE MATHEMATICAL OLYMPIAD
NATIONAL CAPITAL REGION
Qualifying Stage: January 14, 2023


DEPARTMENT OF EDUCATION
DIVISION OF MARINDUQUE
Santa Cruz North District
Landy National High School
25th PHILIPPINE
THEMATICAL OLYMPIA
QUALIFYING STAGE


Welcome Participants, Coaches, and Guests!
25th PHILIPPINE MATHEMATICAL OLYMPIAD



ZAMBOANGA SIBUGAY NATIONAL


25th PHILIPPINE MATHEMATICAL OLYMPIAD
REGION 02

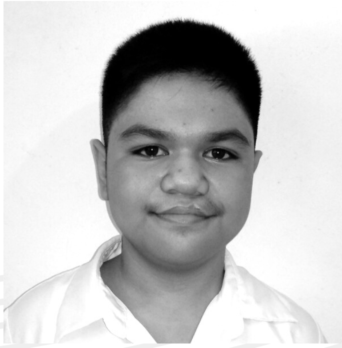

Welcome Participants, Coaches, and Guests!
25th PHILIPPINE MATHEMATICAL OLYMPIAD
Region 12, BARMM & Caraga
Qualifying Stage: January 14, 2023
Area Stage: February 11, 2023
Brought to you by
Host
Official Airline Partner
cebu pacific
Partners
CASIO


Welcome Participants, Coaches, and Guests!
PHILIPPINE
MATHEMATICAL


th PHILIPPINE MATHEMATICAL OLYMPIAD

National Finalists

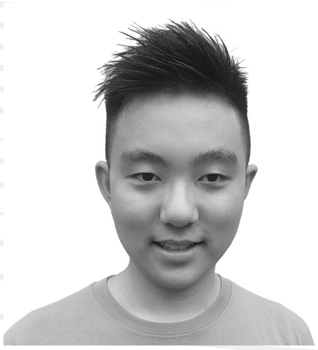
Elliot S. Albano
MGC New Life
Christian Academy



Jonathan D. Anacan
Philippine Science High School
- Western Visayas Campus

Mohammad Nur G. Casib
Philippine Science High School
- Central Mindanao Campus

Enzo Rafael S. Chan
Ateneo de Manila
Senior High School



Enzo Grant D. Ching
Victory Christian
International School

Evgeny D. Cruz
Ateneo de Manila
Senior High School

Raphael Dylan T. Dalida
Philippine Science High School
- Main Campus

Jerson P. Gayagaya
Valenzuela City School of
Mathematics and Science

Benjamin L. Jacob
Philippine Science High School
- Main Campus

Martin Johan M. Ocho
De La Salle University
Integrated School - Manila

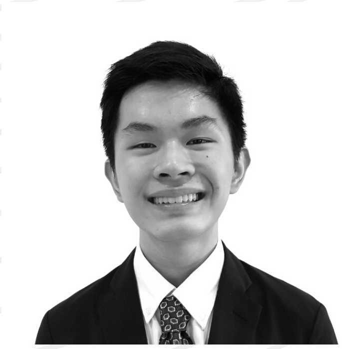
National Finalists

Erich A. Paredes
International School Manila

Alvann Walter W. Paredes Dy
Saint Jude Catholic School

Justin M. Sarmiento
Marcelo H. Del Pilar National High School

Cassidy Kyler L. Tan
Ateneo de Manila Senior High School



Rickson Caleb Y. Tan
MGC New Life Christian Academy

Sean Matthew G. Tan
Jubilee Christian Academy

Jerome Austin N. Te
Jubilee Christian Academy

Kristen Steffi S. Teh
Ateneo de Manila Senior High School

Michael Gerard R. Tongson
Philippine Science High School - Main Campus

Filbert Ephraim S. Wu
Victory Christian International School

25th Philippine Mathematical Olympiad

Qualifying Stage, 14 January 2023

PART I. Choose the best answer. Each correct answer is worth two points.

1. How many four-digit numbers contain the digit 5 or 7 (or both)?

(a) 5416 (b) 5672 (c) 5904 (d) 6416

2. Let $O(0,0)$ and $A(0,1)$. Suppose a point B is chosen (uniformly) at random on the circle $x^2+y^2=1$. What is the probability that OAB is a triangle whose area is at least $\dfrac{1}{4}$?

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{2}{3}$

3. Suppose $a_1 < a_2 < \cdots < a_{25}$ are positive integers such that the average of $a_1, a_2, \ldots, a_{24}$ is one-half the average of $a_1, a_2, \ldots, a_{25}$. What is the minimum possible value of a_{25}?

(a) 26 (b) 275 (c) 299 (d) 325

4. Suppose that a real-valued function $f(x)$ has domain $(-1,1)$. What is the domain of the function $f\left(\dfrac{3-x}{3+x}\right)$?

(a) $(0,+\infty)$ (b) $(-3,3)$ (c) $(-\infty,-3)$ (d) $(-\infty,-3)\cup(3,+\infty)$

5. Aby chooses a positive divisor a of 120 (uniformly) at random. Brian then chooses a positive divisor b of a (uniformly) at random. What is the probability that b is odd?

(a) $\dfrac{2}{5}$ (b) $\dfrac{13}{24}$ (c) $\dfrac{15}{32}$ (d) $\dfrac{25}{48}$

6. Find the sum of the squares of all integers n for which $\sqrt{\dfrac{4n+25}{n-20}}$ is an integer.

(a) 466 (b) 475 (c) 2306 (d) 2531

7. Let $k>1$. The graphs of the functions $f(x)=\log\left(\sqrt{x^2+k^3}+x\right)$ and $g(x)=2\log\left(\sqrt{x^2+k^3}-x\right)$ have a unique point of intersection (a,b). Find $2a$.

(a) $\sqrt{k^3-k+1}$ (b) $k^{3/2}-k^{1/2}+1$ (c) k^2+k+1 (d) k^2-k

8. The sides of a convex quadrilateral have lengths 12 cm, 12 cm, 16 cm, and 16 cm, and they are arranged so that there are no pairs of parallel sides. If one of the diagonals is 20 cm long, and the length of the other diagonal is a rational number, what is the length of the other diagonal?

(a) $\dfrac{48}{5}$ cm (b) $\dfrac{84}{5}$ cm (c) $\dfrac{96}{5}$ cm (d) $\dfrac{108}{5}$ cm

9. How many numbers from 1 to 10^4 can be expressed both as a sum of five consecutive positive integers and as a sum of seven consecutive positive integers, but not as a sum of three consecutive positive integers?

(a) 142 (b) 190 (c) 285 (d) 2096

10. For positive real numbers a and b, the minimum value of

$$\left(18a + \frac{1}{3b}\right)\left(3b + \frac{1}{8a}\right)$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. The value of $m+n$ is

(a) 29 (b) 27 (c) 13 (d) 7

11. In $\triangle ABC$, let D be a point on BC such that $BD : BC = 1 : 3$. Given that $AB = 4$, $AC = 5$, and $AD = 3$, find the area of $\triangle ABD$.

(a) $2\sqrt{3}$ (b) $\sqrt{11}$ (c) $\sqrt{10}$ (d) 3

12. A five-digit perfect square number $\overline{ABCDE}$, with A and D both nonzero, is such that the two-digit number $\overline{DE}$ divides the three-digit number $\overline{ABC}$. If $\overline{DE}$ is also a perfect square, what is the largest possible value of $\overline{ABC}/\overline{DE}$?

(a) 23 (b) 24 (c) 25 (d) 26

13. Consider the sequence $\{a_n\}$, where $a_1 = 1$, and for $n \geq 2$, we have $a_n = n^{a_{n-1}}$. What is the remainder when a_{2022} is divided by 23?

(a) 11 (b) 12 (c) 21 (d) 22

14. How many ways are there to divide a 5×5 square into three rectangles, all of whose sides are integers? Assume that two configurations which are obtained by either a rotation and/or a reflection are considered the same.

(a) 10 (b) 12 (c) 14 (d) 16

15. Let a_1 be a positive integer less than 200. Define a sequence $\{a_n\}$ by $3a_{n+1} - 1 = 2a_n$ for $n \geq 1$. Let A be the set of all indices m such that a_m is an integer but a_{m+1} is not. What is the largest possible element of A?

(a) 5 (b) 6 (c) 7 (d) 8

PART II. All answers should be in simplest form. Each correct answer is worth five points.

1. Let $S = \{1, 2, ..., 2023\}$. Suppose that for every two-element subset of S, we get the positive difference between the two elements. The average of all of these differences can be expressed as a fraction a/b, where a and b are relatively prime integers. Find the sum of the digits of $a + b$.

2. Let x be the number of six-letter words consisting of three vowels and three consonants which can be formed from the letters of the word "ANTIDERIVATIVE". What is $\lfloor x/1000 \rfloor$?

3. Let $f(x) = \cos(2\pi x/3)$. What is the maximum value of $[f(x+1) + f(x+14) + f(x+2023)]^2$?

4. A function $f : \mathbb{N} \cup \{0\} \to \mathbb{N} \cup \{0\}$ is defined by $f(0) = 0$ and $f(n) = 1 + f(n - 3^{\lfloor \log_3 n \rfloor})$ for all integers $n \geq 1$. Find the value of $f(10^4)$.

5. Let $\triangle ABC$ be equilateral with side length 6. Suppose P is a point on the same plane as $\triangle ABC$ satisfying $PB = 2PC$. The smallest possible length of segment PA can be expressed in the form $a + b\sqrt{c}$, where a, b, c are integers, and c is not divisible by any square greater than 1. What is the value of $a + b + c$?

6. In chess, a rook may move any number of squares only either horizontally or vertically. In how many ways can a rook from the bottom left corner of an 8×8 chessboard reach the top right corner in exactly 4 moves? (The rook must not be on the top right corner prior to the 4th move.)

7. In acute triangle ABC, points D and E are the feet of the altitudes from points B and C respectively. Lines BD and CE intersect at point H. The circle with diameter DE again intersects sides AB and AC at points F and G, respectively. Lines FG and AH intersect at point K. Suppose that $BC = 25$, $BD = 20$, and $BE = 7$. The length of AK can be expressed as a/b where a and b are relatively prime positive integers. Find $a - b$.

8. Determine the largest perfect square less than 1000 that cannot be expressed as $\lfloor x \rfloor + \lfloor 2x \rfloor + \lfloor 3x \rfloor + \lfloor 6x \rfloor$ for some positive real number x.

9. A string of three decimal digits is chosen at random. The probability that there exists a perfect cube ending in those three digits can be expressed as a/b, where a and b are relatively prime positive integers. Find $a + b$.

10. Point D is the foot of the altitude from A of an acute triangle ABC to side BC. The perpendicular bisector of BC meets lines AC and AB at E and P, respectively. The line through E parallel to BC meets line DP at X, and lines AX and BE meet at Q. Given that $AX = 14$ and $XQ = 6$, find AP.

Answers

Part I. (2 points each)

1. A
2. D
3. D
4. A
5. D
6. D
7. D
8. C
9. B
10. A
11. B
12. D
13. C
14. B
15. A

Part II. (5 points each)

1. 11 (2024/3)
2. 42
3. 3
4. 8
5. 11 ($2\sqrt{13} - 4$)
6. 532
7. 191 (216/25)
8. 784
9. 301 (101/200)
10. 35

25th Philippine Mathematical Olympiad

Area Stage, 11 February 2023

PART I. The answer to each item is an integer from 1 to 999. No solution is needed. Write your answers in the answer sheet. Each correct answer is worth three points.

1. In parallelogram $WXYZ$, the length of diagonal WY is 15, and the perpendicular distances from W to lines YZ and XY are 9 and 12, respectively. Find the least possible area of the parallelogram.

2. The product of all real numbers x satisfying $x^{2+\log_3(9x)} = \dfrac{2187}{x^2}$ can be written in the form p/q, where p and q are relatively prime positive integers. Find $p+q$.

3. Let x and y be integers satisfying $x^2 + 30x + 25 = y^4$. What is the largest possible value of $x+y$?

4. A game is played on the number line. Initially, there is a token placed at the number 0. In each move, the player can move the token from its current position x, to either $x + 2023$ or $x - 59$. The goal of the game is to move the token to either 1 or -1. What is the minimum number of moves required to achieve this goal?

5. Let x, y, z be three real numbers such that

   - y, x, z form a harmonic sequence; and
   - $3xy, 5yz, 7zx$ form a geometric sequence.

   The numerical value of $\dfrac{y}{z} + \dfrac{z}{y}$ can be expressed in the form p/q, where p and q are relatively prime positive integers. What is $p+q$?

6. There are three novel series Peter wishes to read. Each consists of 4 volumes that must be read in order, but not necessarily one after the other. Let N be the number of ways Peter can finish reading all the volumes. Find the sum of the digits of N. (Assume that he must finish a volume before reading a new one.)

7. Suppose that $P(x)$ and $Q(x)$ are both quadratic polynomials with leading coefficient 1 such that $P(P(x) - x) = (Q(x))^2$ for all real numbers x and $P(2) = 0$. Find the sum of all possible values of $P(10)$.

8. Let S be the sum of all positive integers less than 10^6 which can be expressed as $m! + n!$, where m and n are nonnegative integers. Determine the last three digits of S.

9. How many 9-term sequences $a_1, ..., a_9$ of nonnegative integers are there such that

   - $0 \le a_i < i$ for all $i = 1, ..., 9$; and
   - there are no ordered triples (i, j, k) with $1 \le i < j < k \le 9$, such that a_i, a_j, a_k are all distinct?

10. Suppose $PQRS$ is a convex quadrilateral with $\angle SPQ = \angle PQR = 120^\circ$, $SP - QR = 36$, $RS = 84$, and QR is a positive even integer. Let T be the intersection of lines SP and QR. What is the largest possible perimeter of $\triangle PQT$?

11. In square $ABCD$, P lies on the ray AD past D and lines PC and AB meet at Q. Point X is the foot of the perpendicular from B to DQ, and the circumcircle of triangle APX meets line AB again at Y. Suppose that $DP = \frac{16}{3}$ and $BQ = 27$. The length of BY can be written in the form p/q, where p and q are relatively prime positive integers. Find $p+q$.

12. Seven people are seated together around a circular table. Each one will toss a fair coin. If the coin shows a head, then the person will stand. Otherwise, the person will remain seated. The probability that after all of the tosses, no two adjacent people are both standing, can be written in the form p/q, where p and q are relatively prime positive integers. What is $p+q$?

13. Let a, b, c be real numbers with $1 < a < b < c$ that satisfy the equations

$$\log_a b + \log_b c + \log_c a = 6.5$$

$$\log_b a + \log_c b + \log_a c = 5.$$

Then $\max\{\log_a b, \log_b c, \log_c a\}$ can be written in the form $\sqrt{x} + \sqrt{y}$, where x and y are positive integers. What is $x+y$?

14. In triangle ABC with orthocenter H, $AB = 13$, $BC = 21$ and $CA = 20$. The perpendicular bisector of CH meets BC at P and lines PH and AB meet at Q. The line through Q perpendicular to PQ meets AH at X. The length of AX can be written in the form p/q, where p and q are relatively prime positive integers. Find $p+q$.

15. Bryce has 7 blue socks and 7 red socks mixed in a drawer. He plays a game with Sean. Blindfolded, Bryce takes two socks from the drawer. Sean looks at the socks, and if they have the same color, Sean gives Bryce 1 point. Bryce keeps drawing socks until the drawer is empty, at which time the game ends. The probability that Bryce's score is at most 2 can be written in the form p/q, where p and q are relatively prime positive integers. Find $p+q$.

16. Three circles $\Gamma_1, \Gamma_2, \Gamma_3$ are pairwise externally tangent, with Γ_1, the smallest, having radius 1, and Γ_3, the largest, having radius 25. Let A be the point of tangency of Γ_1 and Γ_2, B be the point of tangency of Γ_2 and Γ_3, and C be the point of tangency of Γ_1 and Γ_3. Suppose now that triangle ABC has circumradius 1 as well. The radius of Γ_2 can then be written in the form p/q, where p and q are relatively prime positive integers. Find the value of the product pq.

17. For each positive integer n, define the function $f_n(x) = |n - x|$. How many real solutions are there to

$$(f_1 \circ f_2 \circ \cdots \circ f_{24} \circ f_{25})(x) = 0?$$

18. Suppose that p is a prime number which divides infinitely many numbers of the form $10^{n!} + 2023$ where n is a positive integer. What is the sum of all possible values of p?

19. Let $p = p_1 p_2 \ldots p_6$ be a permutation of the integers from 1 to 6. For any such permutation p, we count how many integers there are which have nothing bigger on its left. We let $f(p)$ be the number of these integers for the permutation p. For example, $f(612345) = 1$ because only 6 has nothing to its left which is bigger. On the other hand, $f(135462) = 4$ because only 1, 3, 5, and 6 satisfy the condition.

Let S be the sum of $f(p)$ over all the $6!$ different permutations. Find the sum of the digits of S.

20. Determine the sum of all positive integers n for which $2[\tau(n)]^2 = 3n$, where $\tau(n)$ is the number of positive divisors of n.

PART II. Write your solutions to each problem in the solution sheets. Each complete and correct solution is worth ten points.

1. For a set of real numbers A, let $A - A$ be the set of distinct pairwise differences of A; that is,

$$A - A := \{a - b : a, b \in A\}$$

If $|A - A| = 25$, find the sum of all possible values of $|A|$.

2. Let ABC be an acute scalene triangle with orthocenter H. Let M be the midpoint of BC, and suppose that the line through H perpendicular to AM intersects AB and AC at points E and F respectively. Denote by O the circumcenter of triangle AEF, and D the foot of the perpendicular from H to AM. Prove that the line AO intersects the perpendicular from D to BC at a point on the circumcircle of triangle ABC.

Answers to the 25th PMO Area Stage

Part I. (3 points each)

1. 108	**11.** 65 (48/17)
2. 730	**12.** 157 (29/128)
3. 43	**13.** 16
4. 247	**14.** 173 (119/54)
5. 59 (34/25)	**15.** 613 (184/429)
6. 18 (34650)	**16.** 156 (13/12)
7. 64	**17.** 301
8. 130 (4091130)	**18.** 34
9. 503	**19.** 18 (1764)
10. 174	**20.** 96

Part II. (10 points each)

1. For a set of real numbers A, let $A - A$ be the set of distinct pairwise differences of A; that is,

$$A - A := \{a - b : a, b \in A\}$$

If $|A - A| = 25$, find the sum of all possible values of $|A|$.

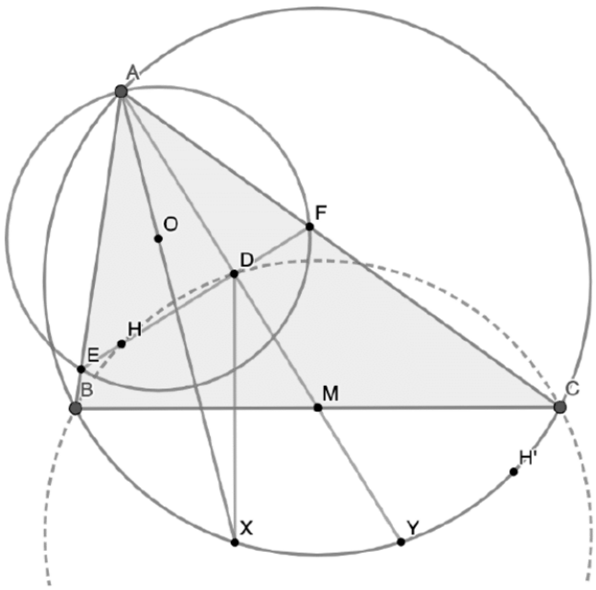
Solution. First, we show that $6 \leq |A| \leq 13$. To do this, we show that if $|A| = n$, then $2n - 1 \leq |A - A| \leq n(n-1) + 1$. The upper bound is easily attained; the number of distinct differences is bounded above by the number of ordered pairs (a, b) of distinct elements of A plus 1, to account for the remaining pairs (a, a) which give a difference of 0; this is exactly $n(n-1)+1$. For the lower bound, suppose that $A = \{a_1, a_2, \ldots, a_n\}$ with $a_1 < a_2 < \cdots < a_n$. Then the $n - 1$ differences $a_n - a_1, a_n - a_2, \ldots, a_n - a_{n-1}$ are all positive and distinct and appear in $A - A$; their additive inverses are all negative and distinct and also appear in $A - A$. Also, $0 \in A - A$ and 0 is not among those differences previously listed. This gives us at least $2(n-1) + 1 = 2n - 1$ differences in $A - A$. These bounds show us that if $|A - A| = 25$, we must have $6 \leq |A| \leq 13$ as desired.

We then show that for any n with $6 \leq n \leq 13$, there exists a set A such that $|A| = n$ and $|A - A| = 25$. Consider now the sets $B = \{1, 2, 4, 8, 12, 13\}$ and $C = \{1, 2, \ldots, 13\}$. Then we have $|B| = 6, |C| = 13$, $B \subseteq C$, and $B - B = C - C = \{-12, -11, \ldots, 0, \ldots, 11, 12\}$. Now, for any set A with $B \subseteq A \subseteq C$, we have $B - B \subseteq A - A \subseteq C - C$, and so we must have $A - A = B - B = C - C$. For any n with $6 \leq n \leq 13$, it is always possible to choose A such that $|A| = n$ and $B \subseteq A \subseteq C$, and this will give us the desired A.

Thus, the sum of all possible values of $|A|$ is $6 + 7 + \cdots + 13 = \boxed{76}$. □

2. Let ABC be an acute scalene triangle with orthocenter H. Let M be the midpoint of BC, and suppose that the line through H perpendicular to AM intersects AB and AC at points E and F respectively. Denote by O the circumcenter of triangle AEF, and D the foot of the perpendicular from H to AM. Prove that the line AO intersects the perpendicular from D to BC at a point on the circumcircle of triangle ABC.

Solution. WLOG assume $AB < AC$. Let AM intersect the circumcircle of ABC again at $Y \neq A$. We first need to prove a claim.



Claim: Quadrilateral $BHDC$ is cyclic.

Proof of Claim: Let AH intersect BC at H_A and EF intersect BC at X. Note that quadrilaterals HH_AMD, $EFBC$, and EH_AMF are cyclic, so by power of a point, we have

$$XH \cdot XD = XH_A \cdot XM = XE \cdot XF = XB \cdot XC,$$

and the claim follows.

We now go back to our original problem. For convenience, let (XYZ) denote the circumcircle of triangle XYZ. Consider the reflection with respect to M. Note that (BHC) maps to (ABC), since the reflection of H is well-known to be the antipode of A with respect to (ABC). It then follows that D maps to Y, so M is the midpoint of DY.

Now let AO intersect (ABC) again at X. Then we note that

$$\angle BAX = \angle EAO = \angle DAF = \angle YAC$$

so $XY||BC$. Now let X' be the reflection of D about BC. Since this maps (BHC) to (ABC), it follows that $X' \in (ABC)$. Let N be the midpoint of DX', which is on BC. Then note that $X'Y||NM$, so $X'Y||BC$, implying $X = X'$. But by construction, $DX = DX'$ is perpendicular to BC, and the desired conclusion follows. $\square$

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