26TH

PHILIPPINE MATHEMATICAL OLYMPIAD

26TH PHILIPPINE MATHEMATICAL OLYMPIAD

Qualifying Stage

December 2, 2023
Various Test Centers Nationwide

Area Stage

January 13, 2024
Various Test Centers Nationwide

National Stage

February 17 and 18, 2024
Department of Mathematics
Ateneo de Manila University

Awarding Ceremony

February 18, 2024
Singson Hall
Ateneo de Manila University

The 26th Philippine Mathematical Olympiad is a project of the Department of Science and Technology - Science Education Institute and the Mathematical Society of the Philippines.

The 26th PMO is also made possible by the support of our partners.

About the PMO

First held in **1984**, the **Philippine Mathematical Olympiad (PMO)** was created as a venue for high school students with interest and talent in mathematics to come together in the spirit of friendly competition and sportsmanship. Its aims are:

1. to stimulate the improvement of mathematics education in the country by awakening greater interest in and appreciation of mathematics among students and teachers, and gaining insights into the levels of mathematical learning;
2. to identify and motivate the mathematically gifted;
3. to identify potential participants to the International Mathematical Olympiad;
4. to provide a vehicle for the professional growth of teachers; and
5. to encourage the involvement of both public and private sectors in the concerted promotion and development of mathematics education.

The PMO is only the first part of the selection program implemented by the Mathematical Society of the Philippine towards the country's participation in the **International Mathematical Olympiad (IMO)**. The twenty national finalists of the PMO will be invited to the **Mathematical Olympiad Summer Camp (MOSC)**, a training program where participants will experience problem solving at a level that will help them grow in mathematical maturity, in preparation for the IMO. The selection tests and quizzes will then determine the six contestants who will form the country's National Team in Mathematics - the Philippine Team to the International Mathematical Olympiad.

The Philippine Mathematical Olympiad, the Mathematical Olympiad Summer Camp, and the country's participation in the International Mathematical Olympiad, are projects of both the **Mathematical Society of the Philippines** and the **Department of Science and Technology - Science Education Institute**.

The PMO this year is the twenty-sixth since 1984. Contestants this year were able to compete onsite (in-person) in 39 test centers nationwide and also online. Last December, **more than 6,000 contestants** from Grades 7 to 12 joined the Qualifying Stage. From this number, **158** joined the Area Stage last January. The **20 National Finalists** were then selected from this pool. We will choose from this group the Philippine Team to the **65th International Mathematical Olympiad** which will be held from July 10-22, 2024 in Bath, United Kingdom.

Message

The Philippines is a nation brimming with inherent talent, exceptional potential, and boundless creativity. It is undeniable that we have made great headways in accelerating our pursuit of establishing a progressive culture of academic discourse by continuously pushing the frontiers of excellence and achieving extraordinary feats here and on the international stage.

Each year, the Philippine Mathematical Olympiad (PMO) brings together some of the nation's brightest high school students to challenge themselves and one another in solving mathematical problems. Mathematics, however, is more than just an academic subject to tackle in school or contests.

Mathematics (Math) occupies a crucial and unique role in society: From carrying out daily human activities to calculating spacecraft trajectories, Math has been an indispensable adjunct to technology and assumed a similar role in the sciences. In other words, it is more than just numbers and formulas - it opens many doors of development and opportunities by creativity and determination. That is why Math is an integral part of general education, most particularly of Science, Technology, Engineering, and Mathematics (STEM).

While creativity is the driving force behind progress, determination is the fuel to keep going. And we, at the Department of Science and Technology-Science Education Institute (DOST-SEI), firmly believe that the Filipino youth have the ingenious flexibility to generate new and original ideas, build upon them, and think out of the box to flexibly work on challenges en route.

The Institute has always trusted in the inherent skills and talent of the Filipino people - most especially the younger generations. And so, trust that we will continue to capitalize on that talent and put a premium on their capabilities by providing the necessary toolkits to better utilize their strengths, cultivate their passion and enthusiasm for Math, and build confidence in their potential. Therefore, giving them the wherewithal to compete and succeed.

We extend our sincerest congratulations, gratitude, and appreciation to the PMO for your passion and unwavering commitment to helping our Filipino talents find and ignite the spark of genius within them to continue to follow the track of upward mobility and success and thrive on the world stage. This dedication alone, for over 25 years, has garnered an astounding number of medals for the Philippines: 4 golds, 19 silvers, 40 bronzes, and 30 honorable mentions.

So, our young Math enthusiasts, may you continue to be a constant source of inspiration and pride yet humble and proficient participants in our STEM engagement endeavors as you unceasingly challenge possibilities and overcome barriers on the international stage. May your creativity and determination give you the spirit to persevere, persist, and succeed in the game until the very end.

Lastly, as we rejoice in the success of this year's PMO, may we always be reminded that beyond winning in the competition, the most essential reward is the power of always outdoing yourself. So I encourage everyone (most especially those who fear Math and the youth) to let your creativity and determination flourish - continue to mirror the Institute's core values and share the same pleasure and courage in the pursuit of solutions, in your own ways.

Our progress as a nation lies in the hands of those who have the opportunity and dedication to show the whole world that, indeed, the Philippines can and will always win.

Thank you.
Mabuhay tayong lahat!

JOSETTE T. BIYO

Director, Science Education Institute
Department of Science and Technology

Message

Congratulations to all the participants of the 26th Philippine Mathematical Olympiad, most especially to the national finalists and their coaches! Congratulations also to the PMO Organizing Team led by Dr. Richard Eden, all the Regional Coordinators, and the Test Development Committee headed by Dr. Christian Paul Chan Shio for successfully putting together another excellent edition of the country's foremost mathematics competition for high school students.

With Dr. Eden at the helm, the PMO has undergone many welcome enhancements, such as recognizing the top junior contestant, the top female contestant, and opening up more test centers nationwide. These enhancements are aimed toward promoting inclusivity in mathematics competitions, and it is my fervent wish to see more females and also more regional competitors making it to the final round.

The Mathematical Society of the Philippines is committed to provide avenues for its members and for the larger Philippine mathematical community to pursue their research interests, disseminate their results, and further enhance their mathematical abilities. This commitment naturally includes the nurturing of young Filipinas and Filipinos, who hopefully will eventually pursue careers in mathematics and its allied fields.

What sets the PMO apart from the other math competitions is the quality of its questions — formulated by seasoned veterans and designed to identify potential members of the Philippine delegation to the International Mathematical Olympiad. The MSP is most certainly pleased with our team's performance in the Asia Pacific and International Mathematical Olympiads in recent years. With the continued support of the DOST-SEI, we at the MSP are confident that our recent accomplishments will be sustained, if not surpassed.
Padayon!

JOSE ERNIE C. LOPE
President
Mathematical Society of the Philippines

Message

Foundation for Upgrading the Standard of Education, Inc.

F·U·S·E

Philippine Council for NGO Certification Accredited
PRC Accredited CPD Provider

Mathematics is normally thought of as just formulas and equations. However, it is pivotal in the particularities of daily living, especially for students – from measuring things, budgeting their allowance and time, participating in games, and much more. More importantly, mathematics develops critical thinking to enable them to be better thinkers and rational beings.

We acknowledge the efforts of all educators serving the Philippine Math Olympiad in developing the mathematical skills of our young Filipinos and their love of numbers, and ultimately, their confidence in themselves. Guiding students toward eligibility to the International Math Olympiad is no mean responsibility – know that as you engage in this commitment, you are accounting your own skills to the Source.

You have received and now, you are giving — congratulations and may you be blessed!

LUCIO C. TAN
Chairman, Board of Trustees
Foundation for Upgrading the
Standard of Education Inc.

Message

We extend our warmest congratulations to the Mathematical Society of the Philippines for the continued success of the Philippine Mathematical Olympiad (PMO) in recent years. The PMO's 26th anniversary highlights the society's unwavering commitment to exceptional mathematical aptitude, inspiring achievers across the nation to reach new heights of success in the future.

To all olympiad participants, C&E Adaptive Learning Solutions (C&E ALS) encourages you to pursue your passion for this challenging yet rewarding field. Together with the Mathematical Society of the Philippines, we are dedicated to nurturing and supporting your talents and abilities as you strive for excellence and open doors to exciting opportunities in the field of mathematics.

JOHN EMYL G. EUGENIO
Chief Operations Officer
C&E Adaptive Learning Solutions

The PMO Team

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Region 10 Paolo Araune
Region 11 Joseph Belida
Region 12, 13, & BARMM Miraluna Herrera

NCR
John Vincent Morales
Chara Deanna Punzal

The PMO Team

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Julius Caadan
Elvira Chua
Maria Montserrat Magdael

The PMO is grateful to these schools which served as test centers.

Region I & CAR
Benguet State University
Don Eulogio de Guzman Memorial National High School
Regional Science High School for Region I
Philippine Science High School - Ilocos Region Campus

Region 2
Cagayan State University
Isabela State University - Echague Campus
Pinaripad National High School

Region 3
Bulacan State University
Central Luzon State University

Region 4A
De La Salle University - Dasmariñas
De La Salle Lipa
University of the Philippines Los Baños
Southern Luzon State University

Region 4B
Western Philippines University - Puerto Princesa
Mindoro State University

Region 5
Ateneo de Naga University
University of Sto. Tomas - Legazpi

Region 6
West Visayas State University
Aklan State University
University of St. La Salle Bacolod
Colegio de la Purisima Concepcion

Region 7
University of the Philippines Cebu
Sisters of Mary School Boystown

Region 8
Leyte Normal University

Region 9
Ateneo de Zamboanga University
Andres Bonifacio College

Region 10
Xavier University - Ateneo de Cagayan
Mindanao State University - Iligan Institute of Technology
Central Mindanao University

Region 11
Ateneo de Davao University

Region 12, 13, and BARMM
Caraga State University
Notre Dame University
Mindanao State University - General Santos
St. Paul University Surigao
Philippine Normal University Mindanao
Albert Einstein School, Inc.

NCR
Ateneo de Manila University
De La Salle University
International School Manila
University of the Philippines Diliman

Awards

TOP CONTESTANT PER REGION
IN THE QUALIFYING STAGE

NCR
Jerome Austin N. Te
Jubilee Christian Academy

Filbert Ephraim S. Wu
Victory Christian International School

Region 1
Gabriel James Valdez
Philippine Science High School - Ilocos Region Campus

CAR
Karl Angelo F. Rigos
Philippine Science High School - Cordillera Administrative Region Campus

Region 2
Khim Marique F. Daquioag
Cagayan National High School - Senior High

Region 3
Justin M. Sarmiento
Marcelo H. Del Pilar National High School

Region 4A
Reuben Joseph R. Felix
Philippine Science High School - CALABARZON Region Campus

Region 4B
Zandrei Killua P. Asi
Oriental Mindoro National High School (Junior High School Department)

Region 5
Joseph Brian C. Monzales
Philippine Science High School - Bicol Region Campus

Region 6
Jonathan D. Anacan
Philippine Science High School - Western Visayas Campus

Region 7
Matthew Andrei C.H. Go
Sacred Heart School - Ateneo de Cebu

Region 8
Bryle Adrian P. Lacabe
Philippine Science High School - Eastern Visayas Campus

Region 9
Josh Chael M. Villanueva
Regional Science High School - IX

Milo M. Quidilla
Zamboanga Chong Hua High School

Region 10
Mohammad Nur G. Casib
Philippine Science High School - Central Mindanao Campus

Region 11
Daniel Day B. Doneza
Davao City National High School

Region 12
Felinwright Niñokyle A. Mesias
Philippine Science High School - SOCCSKSARGEN Region Campus

Region 13
Hans Ethan K. Ting
Philippine Science High School - Caraga Region Campus

BARMM
Aljoharbie S. Liwalug
MSU-Marawi Senior High School

Marif Fatheeya A. Palao
Albert Einstein School, Inc.

Norol-Azah B. Racman
Mindanao State University - University Training Center

Awards

AREA STAGE WINNERS

LUZON

1st **Benedict S. Rodil**
Dasmariñas Integrated High School

2nd **Justin M. Sarmiento**
Marcelo H. Del Pilar National High School

3rd **Reuben Joseph R. Felix**
Philippine Science High School CALABARZON Region Campus

Gabriel James Valdez
Philippine Science High School - Ilocos Region Campus

VISAYAS

1st **Jonathan D. Anacan**
Philippine Science High School - Western Visayas Campus

2nd **Matthew Andrei C.H. Go**
Sacred Heart School - Ateneo de Cebu

3rd **Lance Christopher M. Jimenez**
San Roque College de Cebu

Jose Mari Paolo B. Rollan
City of Bogo Senior High School

MINDANAO

1st **Mohammad Nur G. Casib**
Philippine Science High School - Central Mindanao Campus

2nd **Felinwright Niñokyle A. Mesias**
Philippine Science High School - SOCCSKSARGEN Region Campus

3rd **Hans Ethan K. Ting**
Philippine Science High School - Caraga Region Campus

NCR

1st **Jerome Austin N. Te**
Jubilee Christian Academy

2nd **Filbert Ephraim S. Wu**
Victory Christian International School

3rd **Elliot Xander S. Albano**
MGC New Life Christian Academy

Erich A. Paredes
International School Manila

Awards

SPECIAL AWARDS

The 26th PMO will hand out the following special awards at the Awarding Ceremony.

Top Junior Contestant

This is for the student from Grade 7, Grade 8, or Grade 9 with the highest score in the National Stage. The awardee will receive a medal, certificate, and P 5,000.

Top Female Contestant

This is for the female contestant with the highest score in the National Stage. The awardee will receive a medal, certificate, and P 5,000.

PRIZES FOR PMO WINNERS

The prizes for the three winners of the PMO in the National Stage are the following:

CHAMPION **P 100,000, medal, trophy, certificate**

FIRST RUNNER-UP **P 75,000, medal, trophy, certificate**

SECOND RUNNER-UP **P 50,000, medal, trophy, certificate**

Their coaches will receive P 10,000, P 7,500, and P 5,000, respectively, and a certificate.

Their schools will receive a trophy.

All National Finalists will be invited to the Mathematical Olympiad Summer Camp. The MOSC is the training camp that will select the country's contestants to the 65th International Mathematical Olympiad this July 2024 in Bath, United Kingdom.

National Finalists

Elliot Xander S. Albano
MGC New Life
Christian Academy

Jonathan D. Anacan
Philippine Science High School
- Western Visayas Campus

Ervin Joshua V. Bautista
Southville International
School and Colleges

Kody Briones
Philippine Science High School
- Main Campus

Mohammad Nur G. Casib
Philippine Science High School
- Central Mindanao Campus

Ethan Jared R. Chan
British School Manila

Kei Hang Derek H. Chan
British School Manila

Hans Gabriel De Vera
Philippine Science High School
- Main Campus

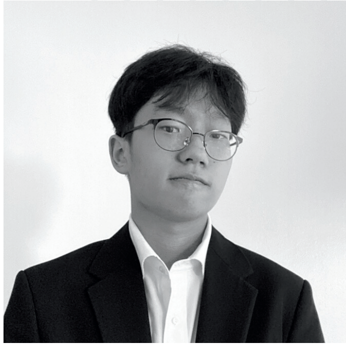
Sofio P. Embalsado III
Ateneo de Manila
Senior High School

Mateo Inigo Espocia
Philippine Science High School
- Main Campus

National Finalists

Neo Angelo G. Gatlabayan
British School Manila

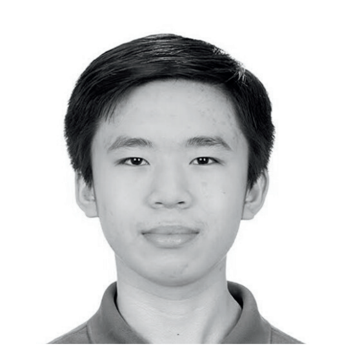
Benjamin Jacob
Philippine Science High School - Main Campus



June Hyung Kim
International School Manila

Patric Xamwell Legaspi
Philippine Science High School - Main Campus

Erich A. Paredes
International School Manila



Alvann Walter W. Paredes Dy
Saint Jude Catholic School

Luke Sebastian C. Sy
Grace Christian College

Jerome Austin N. Te
Jubilee Christian Academy

Zion Skye Earl Carmelo Uy
Philippine Science High School - Main Campus

Filbert Ephraim S. Wu
Victory Christian International School

CONGRATULATIONS!

Philippine Team

to the 64th International Mathematical Olympiad

held in Chiba, Japan last July 2-13, 2023

IMO 2023
Chiba, JAPAN 64th

SILVER MEDALISTS

BRONZE MEDALISTS

DALIDA
Raphael Dylan
Philippine Science High School Main Campus

TE
Jerome Austin
Jubilee Christian Academy

WU
Filbert Ephraim
Victory Christian International School

CASIB
Mohammad Nur
Philippine Science High School Central Mindanao Campus

PAREDES DY
Alvann Walter
Saint Jude Catholic School

TAN
Rickson Caleb
MGC New Life Christian Academy

Team Leader: *Hazel Joy Shi - University of the Philippines Diliman*
Deputy Team Leader: *Kerish Villegas - Ateneo de Manila University*
Trainer: *Russelle Guadalupe - University of the Philippines Diliman*

The Philippines Attains All-Time High Team Score

Top 26 in Country Ranking

3 Silver and 3 Bronze All-Medal Finish

Rank 10th in Problem 5

The participation of the Philippines in the International Mathematical Olympiad is a project of the Department of Science and Technology - Science Education Institute and the Mathematical Society of the Philippines.

(Left to Right) Dr. Randolf Sasota and Dr. Josette Biyo (Director) of DOST-SEI, Casib, Paredes Dy, Wu, Dalida, Te, Tan, Shi, Guadalupe, Villegas


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
BICOL REGION
Qualifying Stage: December 2, 2023
Area Stage: January 13, 2024


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION I and CAR
Qualifying Stage: December 2, 2023
Area Stage: January 13, 2024
Partners
CASIO


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION 6 (Western Visayas)
Qualifying Stage: December 2, 2023
Area Stage: January 13, 2023
CASIO


Welcome Participants, Coaches, and
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
CARAGA REGION


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION IV-B MIMAROPA
Kishen9


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION 11
Qualifying Stage: December 2, 2023
Area Stage: January 13, 2024


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION 6 (Western Visayas)
Qualifying Stage: December 2, 2023
Area Stage: January 13, 2024
Host
Partners
CASIO


Welcome Participants, Coach
and Guests!
PHILIPPINE


COLLEGE OF
SCIENCE AND
MATHEMATICS


Welcome Participants, Coaches, and Guests!
PHILIPPINE
MATHEMATICAL
OLYMPIAD


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION III
Qualifying Stage: December 02, 2023
Brought to you by
Host
Partners
CASIO


Ateneo de Zamboanga University
Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION IX

Welcome Participants, Coaches, and Guests!
PHILIPPINE
MATHEMATICAL

Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
Region VIII - Eastern Visayas
Qualifying Stage: December 2, 2023
Academic Building, Leyte Normal University

CASIO
Math Campus



Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION III

PHILIPPINE
MATHEMATICAL
OLYMPIAD


JUNIOR HIGH SCHOOL
Welcome Participants, Coaches, and Guests!


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
National Capital Region
Qualifying Stage: December 2, 2023
Area Stage: January 13, 2024
CASIO


Welcome Participants, Coaches, and Guests!
26TH PHILIPPINE MATHEMATICAL OLYMPIAD
REGION VII
Qualifying Stage: December 02, 2023
Area Stage: January 13, 2023
Brought to you by
Host
Partners
CASIO


Welcome Participants, Coaches, and Guests
PHILIPPINE MATHEMATICAL OLYMPIAD
REGION VII


Welcome Participants, Coaches, and Guests!
PHILIPPINE MATHEMATICAL OLYMPIAD

26th Philippine Mathematical Olympiad

Qualifying Stage, 02 December 2023

PART I. Choose the best answer. Each correct answer is worth one point.

1. Let a, b, c be positive integers such that

$$3.14 = a + \cfrac{1}{b + \frac{1}{c}}.$$

What is $a + b + c$?

(a) 11 (b) 13 (c) 17 (d) 19

2. What is the area of a rhombus whose diagonals have lengths 14 and 48, respectively?

(a) 100 (b) 336 (c) 625 (d) 672

3. The arithmetic mean of 11 integers is 10. After adding 20 to each of the first four and subtracting 24 from each of the last seven, what is the new mean?

(a) 2 (b) 3 (c) 4 (d) 6

4. Let a and b be the last two digits of the 5-digit number $\overline{764ab}$. What is the largest possible value of the product of ab^2 if the 5-digit number is divisible by 6?

(a) 576 (b) 567 (c) 512 (d) 441

5. An urn contains two white and two black balls. John draws two balls simultaneously from the urn. If the balls are of different colors, he stops. Otherwise, he returns both balls to the urn and then repeats the process. What is the probability that he stops after exactly three draws?

(a) $\frac{1}{16}$ (b) $\frac{2}{27}$ (c) $\frac{1}{8}$ (d) $\frac{4}{27}$

PART II. Choose the best answer. Each correct answer is worth two points.

1. Today — the 2nd day of the 12th month of the year 2023 — marks the start of the 26th Philippine Mathematical Olympiad. What is the remainder when 2023^{12^2} is divided by 26?

(a) 1 (b) 2 (c) 3 (d) 4

2. There are 2024 coins laid out in a row. For some positive integer x, at least $\frac{2}{3}$ of the first x coins are heads, and at least $\frac{4}{5}$ of the last x coins are tails. What is the maximum possible value of x?

(a) 1012 (b) 1079 (c) 1288 (d) 1380

3. Find the remainder when $\sum_{n=1}^{2023} 2023^n$ is divided by 15.

(a) 4 (b) 7 (c) 10 (d) 13

4. The integers from 1 to 2023 are written on a long blackboard in a straight line. Millie colors any number divisible by 23 in blue, any number divisible by 88 in yellow, any number divisible by both 23 and 88 in green, and any number not divisible by either in red. How many adjacent pairs of numbers have different colors?

(a) 222 (b) 220 (c) 218 (d) 216

5. Let r and s be the roots of the polynomial $x^2 + 2x + 3$. The value of $\frac{1}{r^2 - 1} + \frac{1}{s^2 - 1}$ is

(a) $-\frac{1}{5}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{5}$ (d) 1

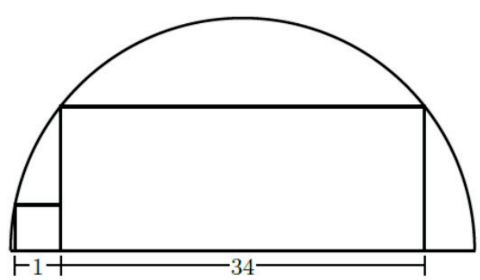
6. Eight people are to sit around a round table on equally-spaced seats. Two of the eight people, Alice and Bob, insist on sitting next to each other. Meanwhile, another two, Clara and Dan, insist on sitting opposite each other. How many ways are there to seat the eight people? Rotations are considered equivalent.

(a) 96 (b) 144 (c) 192 (d) 240

7. A finite sequence $a_1, a_2, \ldots, a_n$, with $n \geq 3$, is said to be *strongly unimodal* if there exists an integer k, with $1 < k < n$, such that $a_1 < \cdots < a_k > a_{k+1} > \cdots > a_n$. How many strongly unimodal sequences $a_1, \ldots, a_{26}$ are there which are permutations of the set $\{1, 2, \ldots, 26\}$?

(a) $2^{25} - 2$ (b) $2^{25} - 1$ (c) $2^{26} - 2$ (d) $2^{26} - 1$

8. A square of side length 1 and a rectangle of length 34 are inscribed in a semicircle as shown below. What is the area of the rectangle?



(a) 204 (b) 238 (c) 272 (d) 306

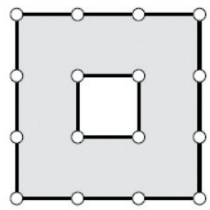
9. A palindrome is a number that reads the same backward and forward. If a palindrome between 100 and 1000 (inclusive) is chosen uniformly at random, what is the probability that this number is divisible by 11?

(a) $\frac{1}{15}$ (b) $\frac{4}{45}$ (c) $\frac{8}{45}$ (d) $\frac{1}{5}$

10. Ernest has a jar with 26 identical cookies. He wants to finish the cookies in the jar by eating 2 or 3 cookies each day. In how many ways can he do this?

(a) 616 (b) 619 (c) 622 (d) 625

11. Below is a drawing of the lattice polygon P:



It has 16 integer points (i.e., points with integer coordinates) on its boundary (indicated by circles), and none in its interior (shaded light gray above).

The polygon $26P$ is obtained by dilating P by a factor of 26. That is, it consists of the points $(26x, 26y)$ wherein (x, y) is a point in the interior or on the boundary of P.

How many integer points are in the interior of $26P$?

(a) 5100 (b) 5200 (c) 5512 (d) 5616

12. The real numbers x, y are such that $x \neq y$ and

$$\frac{x}{26 - x^2} = \frac{y}{26 - y^2} = \frac{xy}{26 - (xy)^2}.$$

What is $x^2 + y^2$?

(a) 626 (b) 650 (c) 677 (d) 729

13. Three lines with slope 1, 7, and c intersect to form a nondegenerate isosceles triangle. If $1 < c < 7$, what is c?

(a) $\frac{8}{7}$ (b) $\frac{13}{7}$ (c) 2 (d) $\frac{15}{7}$

14. Let a_n be a sequence defined by $a_1 = 17$ and $a_{n+1} = \dfrac{2023a_n}{2023 - 2a_n}$. How many terms in the sequence are integers?

(a) 4 (b) 6 (c) 8 (d) 10

15. Let $ABCD$ be a square. Point E is in its interior such that $\angle AED = 90^\circ$ and $\angle EBC = \angle EAB + 45^\circ$. If $AE = 9\sqrt{2}$, find BE.

(a) 9 (b) $9\sqrt{2}$ (c) 18 (d) $18\sqrt{2}$

PART III. The answer to each item is an integer from 1 to 999. Each correct answer is worth five points.

1. What is the largest prime number which divides

$$\frac{2 + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4! + \cdots + 2023 \cdot 2023!}{2022!}?$$

2. Tasyo is thinking of a two digit number. He tells Basilio the tens digit and Crispin the ones digit, and challenges them to guess the number. Their conversation goes as follows:

Crispin: I know the number is not prime.

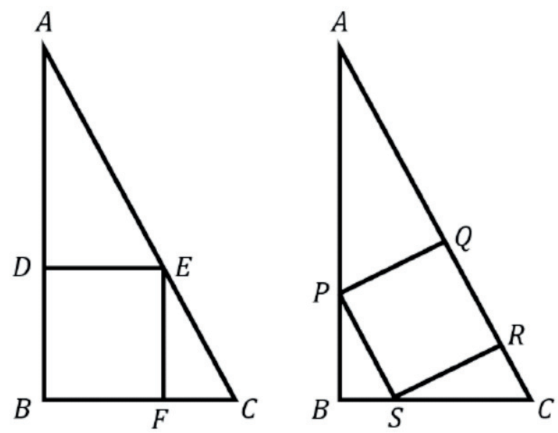
Tasyo: Correct, it is divisible by 3.

Basilio: Now I know what the number is.

What digit did Tasyo tell Crispin?

3. Let $ABCD$ be a square and P be a point on segment AB. Segments CP and BD intersect at Q, and BD is extended beyond B to a point S. Finally, T is the intersection of line CP and the line through S parallel to AB. If $AB = 18$ and $PT = 2CQ$, what is the area of quadrilateral $PBST$?

4. Let ABC be a right triangle with hypotenuse CA. Suppose $AB = 12$ and $BC = 5$. A square is inscribed in this triangle in two different ways, as shown in the figure below.



The ratio of the length of PQ to that of DE can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

5. For a nonnegative integer n, we define $S(n)$ to be the sum of the decimal digits of n. Define $f(N) = \sum_{n=1}^{N} |S(n) - S(n-1)|$. How many positive integers less than or equal to 2024 are **not** equal to $f(N)$ for some N?

6. Let x_1, x_2, x_3 be real numbers such that $x_k > -2k$, for $k = 1, 2, 3$ and $x_1 + 3x_2 + 5x_3 \leq 28$. The minimum value of

$$\frac{1}{x_1 + 2} + \frac{3}{x_2 + 4} + \frac{5}{x_3 + 6}.$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

7. For positive integers i, j, k, ℓ less than 5, let $A_{ijk\ell}$ be an integer. Suppose that the integers $A_{ijk\ell}$ satisfy the following properties:

(a) $A_{ijk\ell} = -A_{jik\ell}$;

(b) $A_{ijk\ell} = -A_{ij\ell k}$;

(c) $A_{ijk\ell} = +A_{k\ell ij}$.

What is the maximum number of distinct integers $A_{ijk\ell}$?

8. If p and q are prime numbers such that p divides $26q - 1$ and q divides $26p + 1$, find the sum of all possible values of $p + q$.

Answers to the 26th PMO Qualifying Stage

Part I. (1 point each)

1. C
2. B
3. A
4. C
5. B

Part II. (2 points each)

1. A
2. D
3. A
4. D
5. B
6. C
7. A
8. A
9. B
10. A
11. B
12. C
13. C
14. D
15. C

Part III. (5 points each)

1. 23
2. 4
3. 648
4. 450 (221/229)
5. 895
6. 17 (9/8)
7. 43
8. 398

26th Philippine Mathematical Olympiad

Area Stage, 13 January 2024

PART I. The answer to each item is an integer from 1 to 999. No solution is needed. Write your answers in the answer sheet. Each correct answer is worth three points.

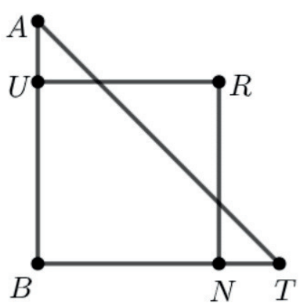
1. Let a and b be relatively prime integers such that $a-b$ and $a+b$ are *not* relatively prime. What is the greatest common divisor of $a-b$ and $a+b$?

2. In the figure below, the square $BURN$ and the isosceles triangle BAT have the same area. What is the measure of $\angle ART$, in degrees?

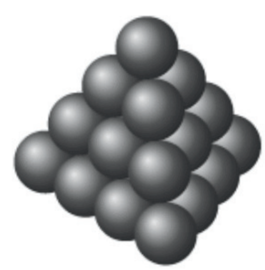
3. The Department of Education of the Philippines orders public schools (elementary or secondary) to have class sizes between 15 and 65, inclusive (*Order Number 62, Series of 2004*). Suppose that all public schools comply with this order, and that each possible class size is attained by some class in some public school. What is the largest integer N with the property that there is certainly some class in some public school in the country that has at least N students with the same birth month?

4. How many ordered pairs of positive integers (x, y) are there so that the set $\{x, y, 1, 3, 5\}$ has a unique mode, and the arithmetic mean, median, and mode have the same value?

5. A stack of balls in the shape of a triangular pyramid contains 2024 balls. How many layers does it have? (Below is an example of a triangular pyramid with 4 layers.)



For Problem 2



For Problem 5

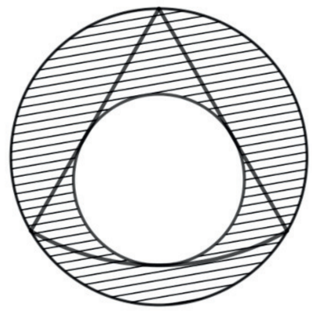
6. What is the greatest number of checkers that can be placed on the squares of a 26×26 checkerboard so that each unit square contains at most one checker, each checker lies entirely inside one unit square, and no four of them lie on the same 2×2 square?

7. Let $f(x) = 1 - 1/x$. Suppose that a, b, and c are the roots of the equation

$$x + f(x) + f(f(x)) = 26.$$

Find $a^2 + b^2 + c^2$.

8. A circular sector has central angle $60°$. A smaller circle is inscribed in the sector, tangent to the two radii and the arc; a larger circle is circumscribed about the same sector as shown below. If the area of the shaded region is equal to 2024, find the remainder when the area of the smaller circle is divided by 1000.



9. Let n be the number of ways all of the letters of the word OLYMPIAD can be placed on the squares of a 3×3 tic-tac-toe board (at most one letter per square) such that no two vowels are on the same column and no two vowels are on the same row. What are the first three digits of n? (Recall that A, E, I, O, U are vowels.)

10. Fern writes down all integers from 1 to 999 that do not contain the digit 2, in increasing order. Stark writes down all integers from 1 to 999 that do not contain the digit 6, in increasing order. How many numbers are written in both of their lists, in the same position?

11. For positive integers p, a, b, and q, let $[p, a, b, q]$ be the sum of the first and last digits of $p^a + b^q$. For example, $[2, 10, 10, 3] = 2 + 4 = 6$ since $2^{10} + 10^3 = 1024 + 1000 = 2024$. Consider the values of $[p, a, 10, q]$ for all p, a, q such that $p \leq 10$, $a \leq 10$, $10 \leq q \leq 20$, and the integers p and q are prime numbers. Give the first three digits of the sum of all these values. (Note that if a value of $[p, a, 10, q]$ appears more than once, then it should be considered in the sum as many times as it appears.)

12. Let $P(x)$ be a polynomial of degree 2 and with leading coefficient 1 such that $P(k) \in \{r, s, t\}$ for $k = 1, 2, 3, 4, 5, 6$, and where r, s, t are real numbers satisfying $r+s+t = 2024$. Find the value of $P(0)$.

13. Let n be the smallest positive integer for which there are exactly 2023 positive integer solutions (x, y) to the equation

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{n}.$$

Determine the first three digits of n.

14. For a set T of numbers, we define $S(T)$ to be the sum of the elements of T. We take $S(\varnothing) = 0$. As usual, $|T|$ denotes the number of elements of T. Let A be the set of positive integers less than 1000 whose digits consist of only 1s or 3s (or both). We choose a random subset B of A. The probability that $S(B)$ and $|B|$ end in the same digit can be expressed in the form $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$?

15. How many ways are there to arrange all the digits in the number 629,999,999 so that the resulting number is divisible by 37?

16. In triangle ABC, suppose that $2\angle BAC + \angle ABC = \angle BCA$, $AB = 260$, and $AC = 26$. If BC can be expressed in the form $m\sqrt{n}$, where m and n are integers, and n is square-free, find $m - n$.

17. Let $f : \mathbb{R}\backslash\{0, 1\} \to \mathbb{R}$ be a function such that

$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{1}{x(1-x)}.$$

The fractional part of $f(2024)$ can be expressed in the form $\frac{a}{b}$, where a and b are relatively prime positive integers. Find the remainder when $b - a$ is divided by 1000.

18. If x is an irrational number and k is an integer such that $x^{14} + kx$, $x^3 + 2x^2$, and $x^2 + x$ are all rational numbers, what is the value of k?

19. Himmel has a watch with two hands: a minute hand and an hour hand. Starting at 12:00 pm, he ran 3 laps around a long oval track at a constant rate, and finished before 1:00 pm. After every full lap, he recorded the distance between the tips of the two hands on his watch. In the order they were recorded, these lengths were 11 mm, 16 mm, and 17 mm. The minute hand is longer than the hour hand by $\sqrt{c}$ mm, where c is a positive integer. What is the value of c?

20. There are n points $P_1, P_2, \cdots, P_n$ on the perimeter of a regular octagon $ABCDEFGH$. None of these points is a vertex of the octagon, and the segments $AP_1, AP_2, \cdots, AP_n$ divide the octagon into regions of equal area. Suppose that one of the sides of the octagon contains exacly 4 of the n points, and its opposite side contains exactly 9 of the n points. What is n?

PART II. Write your solutions to each problem in the solution sheets. Each complete and correct solution is worth ten points.

1. Suppose that Eisen can do the following as many times as he wants to a number written on a blackboard: choose two positive integers m and n such that the number on the blackboard is $m \cdot n$, erase this number on the board, and replace it with $m + n$. For example, if 12 is written on the blackboard, Eisen can replace it with 7, because $12 = 3\times 4$ and $7 = 3 + 4$. If the number 2023 is initially written on a blackboard, determine, with proof, all possible numbers that Eisen can write on the blackboard.

2. Determine all positive integers k less than 2024 for which $4n+1$ and $kn+1$ are relatively prime for all integers n.

Answers to the 26th PMO Area Stage

Part I. (3 points each)

1. 2

2. 135

3. 6

4. 3

5. 22

6. 507

7. 630

8. 12 (1012)

9. 259 (25920)

10. 124

11. 976

12. 684

13. 209 $(n = 2^8 \cdot 3^8 \cdot 5^3 = 209,952,000)$

14. 75 $(\frac{11}{64})$

15. 27

16. 68 $(78\sqrt{10})$

17. 11 $(\frac{1013}{2024})$

18. 377

19. 46

20. 54

Part II. (10 points each)

1. The number 2023 is written on a blackboard. Eisen can do the following as many times as he wants: choose two positive integers m and n such that the number on the blackboard is $m \cdot n$, erase the number on the board, and replace it with $m + n$. For example, if 12 is written on the blackboard, Eisen can replace it with 7, because $12 = 3 \times 4$ and $7 = 3 + 4$. Determine, with proof, all possible numbers Eisen can write on the blackboard.

Answer: Any positive integer greater than or equal to 5.

Proof. The solution has two parts:

We first show that all positive integers at least 5 can eventually be written on the blackboard. First, Eisen can write 5 with the following sequence of moves:

- $2023 = 17 \times 119 \implies 17 + 119 = 136$
- $136 = 8 \times 17 \implies 8 + 17 = 25$
- $25 = 5 \times 5 \implies 5 + 5 = 10$
- $10 = 2 \times 5 \implies 2 + 5 = 7$
- $7 = 1 \times 7 \implies 1 + 7 = 8$
- $8 = 2 \times 4 \implies 2 + 4 = 6$
- $6 = 2 \times 3 \implies 2 + 3 = 5$

Finally, by choosing $m = 1$ repeatedly, Eisen can always write the number one higher than the one on the blackboard currently. This shows that Eisen can write any positive integer at least 5.

Alternatively, here is an induction solution that shows Eisen can eventually write 5, if the number on the board is at least 5.

Base case: the number on the blackboard is 5. Eisen does not have to do any move at all.

Inductive step: Suppose that if Eisen can write any positive integer at most k, then he can eventually turn it into 5. We show that if the number on the blackboard is instead $k+1$, Eisen can turn it into a positive integer in the range $[5, k]$; by hypothesis, Eisen can then turn this number into 5.

If $k+1$ is even, then choose $m = 2$ and $n = \frac{k+1}{2}$. Because

$$\frac{k+1}{2} + 2 = \frac{k+5}{2} \leq \frac{k+k}{2} = k,$$

the sum is at most k. Otherwise, first choose $m = 1$ and $n = k+1$, turning this number into $k+2$. Then, we choose $m = 2$ and $n = \frac{k+2}{2}$. Because $k+1$ is odd, k must be even, implying $k \geq 6$. Now as

$$\frac{k+2}{2} + 2 = \frac{k+6}{2} \leq \frac{k+k}{2} = k,$$

the sum is at most k again.

We then end similarly to the first solution, taking $m = 1$ repeatedly.

Second, we show that Eisen cannot write any positive integer less than 5. Because we have shown that Eisen can write any positive integer at least 5, another way to phrase this is that there do not exist two positive integers m and n that satisfy both $mn \geq 5$, and $m+n \leq 4$. But the only pairs (m, n) that satisfy the second condition (up to swapping) are:

- $(1, 1)$ with product 1
- $(1, 2)$ with product 2
- $(1, 3)$ with product 3
- $(2, 2)$ with product 4

None of these products are at least 5, so Eisen cannot turn any integer at least 5 into one less than 5. □

2. Determine all positive integers k less than 2024 for which $4n+1$ and $kn+1$ are both relatively prime for all integers n.

Answer: $k = 2, 3, 5, 6, 8, 12, 20, 36, 68, 132, 260, 516, 1028$

Proof. Let $d = \gcd(4n+1, kn+1)$. Since $4n+1$ is odd, d must be odd. Also, $d \mid 4n+1$ and $d \mid kn+1$ for all integers n, so $d \mid n(k-4)$ for all integers n. If we let $n = d+1$, we get $d \mid k-4$. So if $k - 4 = \pm 2^l$ for some integer $l \geq 0$, then d must be a positive odd divisor of $\pm 2^l$, forcing $d = 1$. Thus, all numbers k of the form 4 ± 2^l less than 2024 are valid, which are $2, 3, 5, 6, 8, 12, 20, 36, 68, 132, 260, 516, 1028$.

Now, we will show all other values of k are invalid. If $k \neq 4 \pm 2^l$ for some integer $l \geq 0$, then $k-4$ must have an odd divisor $t > 1$. Note that $2 \mid t-1, t+1$, so $4 \mid t^2 - 1$. Let $n = (t^2-1)/4$, so $4n+1 = t^2$, which is obviously divisible by t, and $kn+1 = 4n+1+(k-4)n = t^2 + (k-4)n$, which is divisible by t because $k-4$ is divisible by t. Thus, there exists an integer n such that $\gcd(4n+1, kn+1) > 1$, so such k is invalid. □

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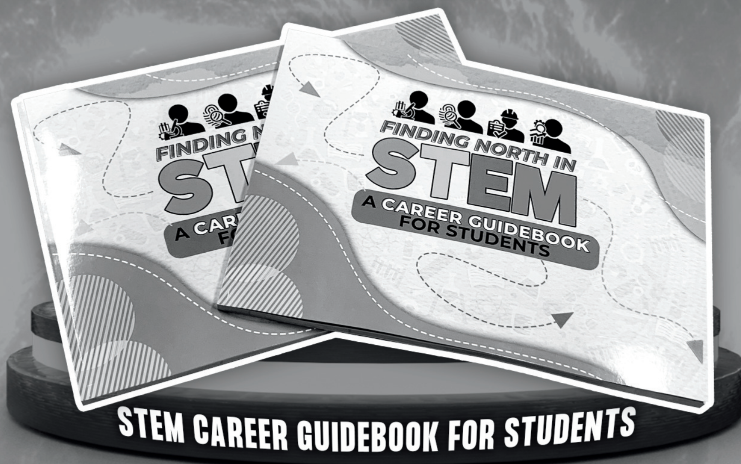

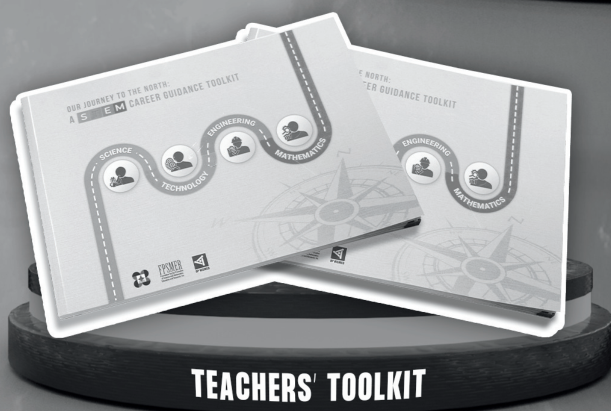

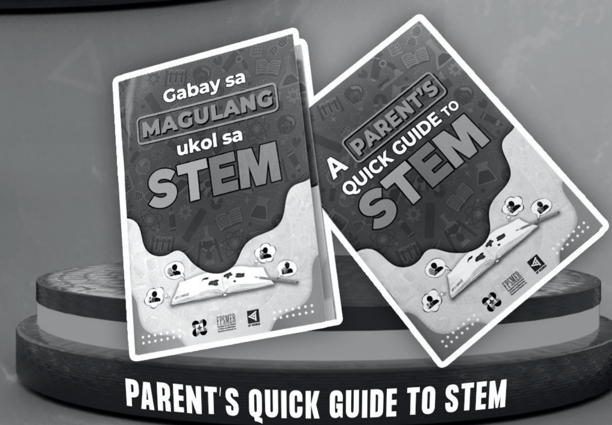

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